Republic Of Iraq
Ministry Of Higher Education and Scientific Research
Misan University
College of Engineering

Mathematical drilling models: A literature review

graduation project is submitted to the Petroleum Engineering Department in partial fulfillment of the requirements for the degree of Bachelor of Science in

College of Engineering - Petroleum Engineering

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Misan, Iraq

2024

بسم الله الرحمن الرحيم

{ يَرْفَعِ اللَّهُ الَّذِينَ آمَنُوا مِنكُمْ وَالَّذِينَ أُوتُوا الْعِلْمَ

دَرَجَاتٍ }

"صدق الله العلي العظيم"

SUPERVISOR CERTIFICATION

I certify that the preparation of this project entitled **Mathematical Drilling Models: A Literature Review,**

prepared by **Mohsen Saleh Abbas, Moumal Rihaan Abdul Hussein, Muntadhir Mohammad Ali, Abdullah Qais Jasib,**

was made under my supervision at the General College of Engineering, Petroleum Engineering Department in partial fulfillment of the Requirements for the Degree of Bachelor of Science in College of Engineering - Petroleum Engineering.

Signature:

Name:

Date:

Dedication

This work is wholeheartedly dedicated to our beloved parents, who have been our source of inspiration and gave us strength when we thought of giving up, who continually provide their moral, spiritual, emotional, and financial support.

To our brothers, sisters, relatives, friends, and classmates who shared their advice and encouragement to finish this work.

To our teachers the guiding lights of our lives.

ACKNOWLEDGMENTS

Firstly, we thank Almighty Allah for the guidance, strength, power of mind, protection, and skills and for giving us a healthy life.

We would like to express our sincere gratitude to our supervisor, **Asst. Lect. Muntadhir Al-Saedi** for his close supervision, helpful remarks, and for considering us, and we would like to reiterate our thanks for our continuous support with all the information, advice, and continuous motivation.

Besides our supervisor, we would like to thank the whole staff of the petroleum engineering department for providing a helping hand and guidance whenever we needed help.

At last, but not least, thanks go to our families. Thanks for instilling in us a strong passion for learning and for doing everything possible to make us successful and thanks for all their sacrifices that make life and its hardships look simply.

ABSTRACT

Minimizing the drilling cost can be achieved by successful modeling of the drilling process and a full understanding of the major factors that affecting directly or indirectly drilling rate. Through the work of numerous investigators, the most important drilling factors have been identified as controllable and uncontrollable factors. This project aims to optimize the drilling process by using many mathematical drilling models that the researchers used in it and study the advantages and limitations of these models. per foot drilled. The most important variables that affect the ROP are i) bit type, ii) formation characteristics, iii) bit operating conditions (i.e., bit type, bit weight, and rotary speed), iv) bit hydraulics, v) drilling fluid properties, vi) bit toot wear. The optimization of the drilling performance, even more in a field development context requires i) data acquisition and ii) data processing. There are three most widely used models for estimating the rate of penetration; i) Maurer, ii) Galle and Woods, and iii) Bourgoyne and Young. To effectively develop an optimal control system for the rotary drilling of oil wells, all the existing mathematical models for calculating the penetration rate would be analyzed critically. The comparative analysis of the drilling rate of penetration models is done with a focus on their application in the petroleum industry, analyzing the parameters of their methods as well as their benefits and weaknesses. A large majority of studies concerning the mathematical modeling of the drilling mud flows have involved various time-independent rheological models such as Bingham, Casson, Herschel–Bulkley, etc.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

1. Introduction

1.1 Overview

Minimizing the drilling cost can be achieved by successful modeling of the drilling process and a full understanding of the major factors that affecting directly or indirectly drilling rate. Through the work of numerous investigators, the most important drilling factors have been identified as controllable and uncontrollable factors. The controllable factors are; bit weight or drilling force, bit rotational speed, bit type and size, hydraulics, and drilling fluid type and properties. While the most important uncontrollable factors are; weather and location, water availability, rig conditions and flexibility, round trip time, rock properties, depth, bottom hole temperature, hole problems, and crew efficiency. The lowest drilling cost doesn't result from increasing penetration rate alone, but also from equipment life and wellbore stability. With a basic understanding of the principal mechanisms and the physical processes involved in the drilling operation, theoretical relationships and empirical correlations based on both field and laboratory measurements materialized out of the 1950s(Hamad-Allah and Ismael, 2008).

Exploration and extraction of oil and gas from the depths of the earth require resistant material and variable diameter drill bits(Wenrui, Jingwei and Bin, 2013). Surface and downhole measurements indicate that drilling systems experience various types of oscillations, and this significantly reduces the rate of drilling penetration and affects the drill bit (wear of drill teeth), drill pipes (pipe deflection), and damaged drilling toolsets(Zare and Shadizadeh, 2014). Oscillations and mechanical vibrations during the drilling of wells and increasing the length of the drilling string cause instability in the operation and increase the possibility of damage to the drilling string system. Drill jump, buckling, and stick-slip phenomena result from axial, lateral, and torsional vibrations entering the drill string system(Tang *et al.,* 2020). Reducing the rate of penetration of the drill, the instability of the lower assembly, causing damage to the drill, and twisting of the drill string are among the harmful results of the vibrations of the drill string, which, in addition to the damage of the rig and the drill string system, also cause an increase in operating costs(Dupriest, 2006). Due to the complex geometric conditions of the drilling string, its torsional resistance is low. On the other hand, torque enters its upper and lower parts simultaneously, and therefore, the drill string experiences significant torsional vibrations(Liu *et al.,* 2020). The study of previous studies shows that one of the most important factors in reducing the penetration rate of the drill and increasing the costs of drilling operations is the stick-slip vibrations that occur due to the friction between the drill bit and the rock. This phenomenon, which results from torsional vibrations, causes the most damage to the drilling string(Sadeghi, Arkan and Özbek, 2022). Another influential factor in the vibrations of the drilling structure is the interaction between the drill bit and the rock, which has been discussed and investigated in numerous articles and studies(Zhu, Tang and Yang, 2014; Tian and Detournay, 2021). A drill string system includes a driver, interface, stabilizer, and rock breaker, known in the drilling industry as a rotary table, drill pipe, and lower assembly. The function of this structure is that on the surface of the earth and with the help of the drilling rig, energy is transferred to the rotary table in the form of torque, and the rotation of the table causes the rotation of the drill pipes and the drill, and at the same time axial force is applied from the surface to the depth. This phenomenon, which results from torsional vibrations, occurs when the drill stops for some reason despite the rotation of the rotary table(Terrand-Jeanne and Martins, 2016; Terrand-Jeanne, Martins and Andrieu, 2018). On the other hand, by applying axial vibrations in addition to torsional vibrations, drilling operations become complicated, and the pair of axial-torsional vibrations with the drill-rock interaction makes it difficult to analyze the system's stability(Besselink *et al.*, 2015; Aarsnes and van de Wouw, 2019).

Several drilling models were proposed in the past to explain the effects of drilling parameters, environment, and geology effect on the ROP. Significant research started at the end of the first half of the 20th Century. The first models were based on R-W-N (ROP, WOB, and RPM) equations, mainly driven by empirical exponents multiplied by proportionality constants to take influencing variables into account, but, later, laboratory tests revealed that R-W-N equations showed reliable results only in the case of perfect hole cleaning conditions. The evolution of these modeling started basically in 1960 with Cunningham (1960), followed by a chain of changes and further research addressed by Maurer (1962), Galle and Woods (1964), Bingham (1965), Bourgoyne Jr. And Young Jr. (1974), and Warren (1981), actual and subsequent models were mainly based on further improvements on top of just-mentioned root ones (Cunningham, 1960; Maurer, 1962; Galle and Woods, 1963; Bingham, 1965; Bourgoyne Jr and Young Jr, 1974; Warren, 1981).

There are many mathematical models in the oil drilling field such as modeling the dynamics of the drill string, modeling the flow of drilling fluid, modeling rock crushing with a drill bit, modeling the mass and heat transfer between the drilling fluid and surrounding soil, Mult physical Model of the Drilling Process (see Fig.1) (Churilova, Lupuleac and Shaposhnikov, 2022).

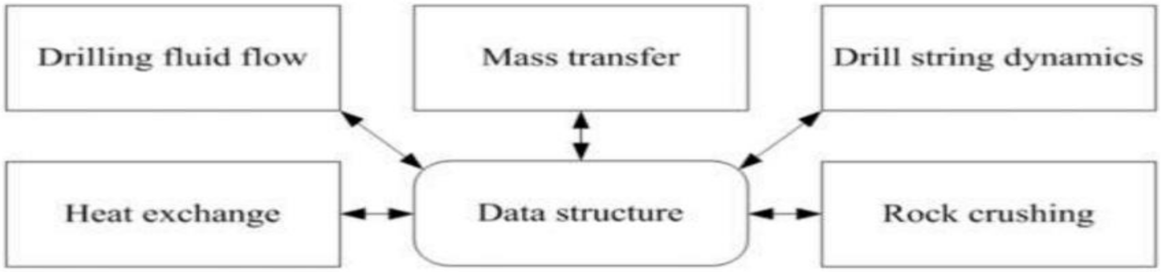

Fig 1. The combined model structure.

The problem of modeling the process of well drilling is a classic Multiphysics problem when it is required to simulate several interrelated processes of different physical natures simultaneously. Note that each of the processes listed above is quite complex, even if considered in isolation from the others. An advanced mathematical model is required to describe each of them.

There are dozens of commercial software packages that simulate one of these processes including Olia soft Well Design software for designing the well (trajectory, casing, etc.) (www.oliasof.com, no date); TADPRO software from Pegasus Vertex Inc. for torque and drag calculation; CTEMP for wellbore circulating temperature prediction; CDEx for casing design; HYDPRO for drilling hydraulics modeling (*www.pvisoftware.com*, no date); software from Halliburton Landmark (*www.halliburton.com*, no date); Drill Plan for digital well construction planning and OLGA for prediction of key operational conditions from Schlumberger (*www.slb.com*, no date); and many others.

The directional drilling process results from the interaction between the BHA and the borehole. The evolution of the borehole geometry is a consequence of bit kinematics. Meanwhile, the penetration of the bit into the rock formation depends on the forces transmitted by the BHA to the bit, which themselves are related to the behavior of the BHA constrained to deform within the borehole. In this respect, the model describing this evolution consists of the association between (i) a model of the near-bit region of the drill string, (ii) a model of the bit/rock interaction, and (iii) kinematic relationships relating the motion of the bit into the rock to geometric variables for the borehole evolution. (Fig. 2) (Detournay, 2009, 2010).

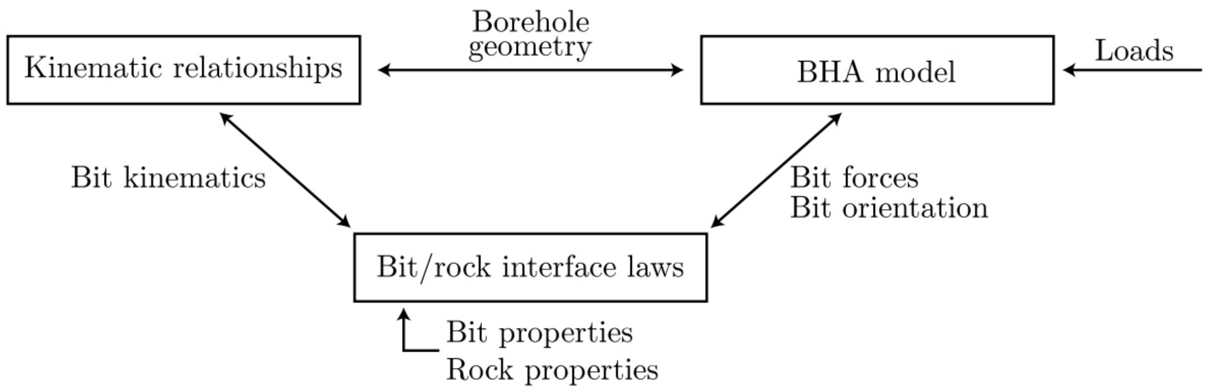

Fig 2: Three elements of the model.

1.2 statement of problem

To provide a drilling process with lower cost, the researchers and engineers need to optimize the drilling process, improving the existing and inventing new equipment, increasing its reliability and fault tolerance, and preventing emergencies.

Currently, drilling the geological exploration wells and the production wells for oil and gas is a complex and expensive process. Carrying out full-size physical experiments is difficult and expensive.

If the drilling process hasn't been optimized, that leads to financial pain caused by increased costs, missed targets, and wasted materials. Environmental harm from more emissions, water pollution, and land degradation. Safety risks increase with higher accident risk for workers and communities. Regulatory trouble is caused by fines for non-compliance with safety and environmental rules. Reduced efficiency: lower oil production and potential damage to the reservoir.

1.3 aim of the project

This project aims to optimize the drilling process by using many mathematical drilling models that the researchers used in it and study the advantages and limitations of these models.

2. Theoretical background

2.1 Factors Affecting Rate of Penetration

2.1.1 Introduction

Factors that affect the ROP are numerous and perhaps important variables. These variables are not recognized well up to date. A rigorous analysis of ROP is complicated by the difficulty of completely isolating the variables under study. For example, the interpretation of field data may involve uncertainties due to the possibility of undetected changes in rock properties. Studies of drilling fluid effects are always plagued by the difficulty of preparing two muds having all properties identical except one which is under observation. While it is generally desirable to increase the penetration rate, such gains must not be made at the expense of overcompensating detrimental effects. The fastest ROP does not necessarily result in the lowest cost per foot of drilled hole. Other factors such as accelerated bit wear, equipment failure, etc., may raise the cost. The factors that affect ROP are listed under two general classifications such as environmental and controllable. Table 1 shows the list of parameters based on these two categories. Environmental factors such as formation properties and drilling fluids requirements are not controllable.

Environmental factors	Controllable factors (alterable)
Depth	Bit Wear State
Formation properties	Bit design
Mud type	Weight on bit
Mud density	Rotary speed
Other mud properties	Flow rate
Overbalance mud pressure	Bit hydraulic
Bottom-hole mud pressure	Bit nozzle size
Bit size	Motor/turbine geometry

Table 1 Variables that affect ROP.

Controllable factors such as weight on bit, rotary speed, and bit hydraulics on the other hand are the factors that can be instantly changed. Drilling fluid is considered to be an environmental factor because a certain amount of density is required to obtain a specific objective to have enough overpressure so that it can avoid the flow of formation fluids. Another important factor is the effect of overall hydraulics on the whole drilling operation. This operation is influenced by many factors such as lithology, type of the bit, downhole pressure and temperature conditions, drilling parameters, and mainly the rheological properties of the drilling fluid. ROP performance is a function of controllable and environmental factors. It has been observed that ROP generally increases with decreased equivalent circulating density (ECD).

Another important term controlling the ROP is the cuttings transport. Ozbayoglu et al. (2004) conducted extensive sensitivity analysis on cuttings transport for the effects of major drilling parameters while drilling for horizontal and highly inclined wells. It was concluded that the average annular fluid velocity is the dominating parameter on cuttings transport, the higher the flow rate the lesser the cuttings bed development. ROP and wellbore inclinations beyond 70° did not have any effect on the thickness of the cuttings bed development. Drilling fluid density has moderate effects on cutting bed development with a reduction in bed removal with increased viscosities. Increased eccentricity positively affected cuttings bed removal. The smaller the cuttings the more difficult it is to remove the cuttings bed. Turbulent flow is better for bed development prevention(Ozbayoglu *et al.*, 2004).

However, in any engineering study of rotary drilling, it is convenient to divide the factors that affect the ROP into the following list:

i) personal efficiency;

ii) rig efficiency;

iii) formation characteristics (e.g., strength, hardness, and/or abrasiveness, state of underground formations stress, elasticity, stickiness or balling tendency, fluid content and interstitial pressure, porosity and permeability, etc.);

iv) mechanical factors (e.g., bit operating conditions – a) bit type, and b) rotary speed, and c) weight on bit);

v) hydraulic factor (e.g., jet velocity, bottom-hole cleaning);

vi) drilling fluid properties (e.g., mud weight, viscosity, filtrate loss, and solid content);

vii) bit tooth wear, and depth.

However, for horizontal and inclined well bores, hole cleaning is also a major factor influencing the ROP. The basic interactive effects between these variables were determined by design experiments. Variable interaction exists when the simultaneous increase of two or more variables does not produce an additive effect as compared with the individual effects. The rate of penetration achieved with the bit as well as the rate of bit wear, has an obvious and direct bearing on the cost per foot drilled.

The most important variables that affect the ROP are:

i) bit type,

ii) formation characteristics,

iii) bit operating conditions (i.e., bit type, bit weight, and rotary speed),

iv) bit hydraulics,

v) drilling fluid properties,

vi) bit toot wear(Hossain and Islam, 2018).

2.1.2 Personal Efficiency: The manpower skills and experiences are referred to as personal efficiency. Given equal conditions during drilling/completion operations, personnel are the key to the success or failure of those operations and ROP is one of them. Overall, well costs as a result of any drilling/completion problem can be extremely high. Therefore, continuing education and training for personnel is essential to have successful ROP and drilling/completion practices(Hossain and Al-Majed, 2015).

2.1.3 Rig Efficiency: The integrity of the drilling rig and its equipment, and maintenance are major factors in ROP and in minimizing drilling problems. Proper rig hydraulics (e.g., pump power) for efficient bottom and annular hole cleaning, proper hoisting power for efficient tripping out, proper derrick design loads, drilling line tension load to allow safe overpull in case of a sticking problem, and well-control systems (e.g., ram preventers, annular preventers, internal preventers) that allow kick control under any kick situation are all necessary for reducing drilling problems and optimization of ROP. Proper monitoring and recording systems that monitor trend changes in all drilling parameters are very important to rig efficiency. These systems can retrieve drilling data at a later date. Proper tubular hardware specifically suited to accommodate all anticipated drilling conditions, and effective mud-handling and maintenance equipment will ensure that the mud properties are designed for their intended functions(Hossain and Al-Majed, 2015).

2.1.4 Formation Characteristics: The formation characteristics are some of the most important parameters that influence the rate of penetration. The following formation characteristics affect the ROP – i) elasticity i.e. elastic limit, ii) ultimate strength, iii) hardness and/or abrasiveness, iv) state of underground formations stress, v) stickiness or balling tendency, vi) fluid content and interstitial pressure, and vii) porosity and permeability. Among these parameters, the most important formation characteristics that affect the ROP are the elastic limit and ultimate strength of the formation. The shear strength predicted by the Mohr failure criteria sometimes is used to characterize the strength of the formation. To determine the shear strength from a single compression test, an average angle of internal friction varies from about 30 to 40° from most rock. The following model has been used for a standard compression test(Gatlin, 1960):

$$\tau_0 = \frac{\sigma_1}{2}\cos\theta \qquad (1)$$

Here, τ_0 = shear stress at failure, psi

σ_1 = compressive stress, psi

θ = angle of internal friction

The threshold force or bit weight $\left(\frac{W}{d}\right)$ required to initiate drilling was obtained by plotting the drilling rate as a function of bit weight per bit diameter and then extrapolating back to a zero-drilling rate. The laboratory correlation obtained in this manner is shown in Figure 3.

The other factors such as the permeability of the formation have a significant effect on the ROP. In permeable rocks, the drilling fluid filtrate can move into the rock ahead of the bit and equalize the pressure differential acting on the chips formed beneath each tooth. Formation as nearly an independent or uncontrollable variable is influenced to a certain extent by hydrostatic pressure. Laboratory experiments indicate that in some formations increased hydrostatic pressure increases the formation's hardness or reduces its drill-ability. The mineral composition of the rock also has some effect on ROP. Rocks containing hard, abrasive minerals can cause rapid dulling of the bit teeth. Rocks containing gummy clay minerals can cause the bit to ball up and drill in a very inefficient manner(Hossain and Al-Majed, 2015).

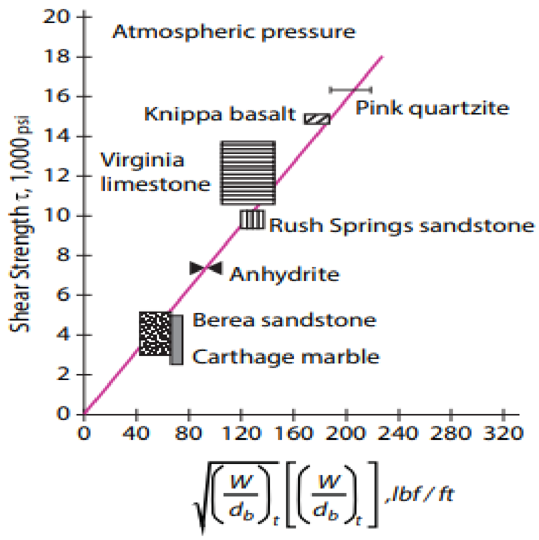

Fig 3 Relationship between rock shear strength and threshold bit weight at atmospheric pressure(Mitchell and Miska, 2011)

2.1.5 Mechanical Factors

The mechanical factors are also sometimes called bit operating conditions. The following mechanical factors affect the ROP – i) bit type, ii) rotary speed, and iii) weight on bit.

i) Bit Type: The bit type selection has a significant effect on the rate of penetration. For rolling cutter bits, the initial penetration rates for shallow depths are often highest when using bits with long teeth and a large cone offset angle. However, these bits are practical only in soft formations because of rapid tooth wear and sudden decline in penetration rate in harder formations(Chen, Dahlem and Dennis, 2001).

The lowest cost per foot drilled usually is obtained when using the longest tooth bit that will give a tooth life consistent with the bearing life at optimum bit operating conditions. The diamond and PDC bits are designed for a given penetration per revolution by the selection of the size and number of diamonds or PDC blanks. The width and number of cutters can be used to compute the effective number of blades. The length of the cutters projecting from the face of the bit (less the bottom clearance) can limit the depth of the cut. The PDC bits perform best in soft, fi rm, and medium-hard, nonabrasive formations that are not gummy. Therefore, the bit type selection i.e., whether a drag bit, diamond bit, or roller cutter bit must be used and the various tooth structures affect to some extent the drilling rate obtainable in a given formation must be considered.

Figure 4 shows the characteristic shape of a typical plot of ROP vs. WOB obtained experimentally where all other drilling variables remain constant. No significant penetration rate

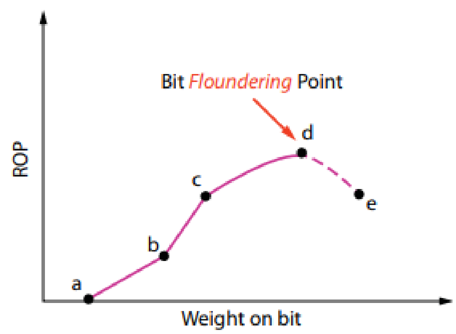

Fig 4 Typical response of ROP to increasing bit weight(Mitchell and Miska, 2011).

is obtained until the threshold bit weight is exceeded (point a). ROP increases gradually and linearly with increasing values of bit weight for low-to-moderate values of bit weight (segment ab). A linear sharp increase curve is again observed at the high bit weight (segment bc). Although the ROP vs. WOB correlations for the discussed segments (ab and bc) are both positive, segment bc has a much steeper slope, representing increased drilling efficiency. Point b is the transition point where the rock failure mode changes from scraping or grinding to shearing. Beyond point c, subsequent increases in bit weight cause only slight improvements in ROP (segment cd). In some cases, a decrease in ROP is observed at extremely high values of bit weight (segment de). This type of behavior sometimes is called bit floundering (point d – bit floundering point). The poor response of ROP at high WOB values is usually attributed to less efficient hole cleaning because of a higher rate of cuttings generation, or because of complete penetration of a bit’s cutting elements into the formation being drilled, without room or clearance for fluid bypass(Gatlin, 1960).

ii) Rotary Speed: Figure 5 shows a characteristic shape typical response of ROP vs. rotary speed obtained experimentally where all other drilling variables remain constant. Penetration rate usually increases linearly with increasing values of rotary speed (N) at low values of rotary speed (segment ab). At higher values of rotary speed (after point b, segment b to c), the rate of increase in ROP diminishes. The poor response of penetration rate at high values of rotary speed usually is also attributed to less efficient bottomhole cleaning. Here, the bit floundering is due to less efficient bottom hole cleaning of the drill cuttings(Mitchell and Miska, 2011).

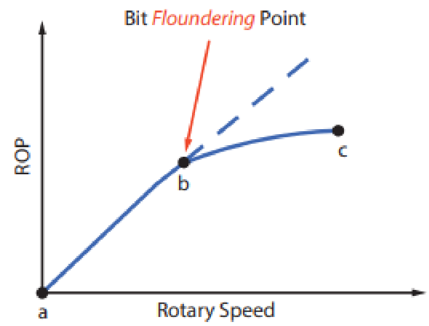

Fig 5 Typical response of ROP to increasing rotary speed (Mitchell and Miska, 2011).

Combined Effect of Bit Weight and Rotary Speed on ROP

Maurer (1962) developed a theoretical equation for rolling cutter bits relating ROP to bit weight, rotary speed, bit size, and rock strength. The equation was derived from the following observations made in single-insert impact experiments – i) the crater volume is proportional to the square of the depth of cutter penetration, ii) the depth of cutter penetration is inversely proportional to the rock strength. For these conditions, the equation can be written as(Maurer, 1962b):

$$ROP = \frac{K}{S_c^2}\left[\frac{W_b}{d_b} - \left(\frac{W_{tb}}{d_b}\right)_t\right]^2 N \qquad (2)$$

Here,

ROP = rate of penetration, ft/min

K = constant of proportionality

Sc = compressive strength of the rock

Wb = bit weight

Wtb = threshold bit weight

db = bit diameter

N = rotary speed

$\left(\frac{W_o}{d_b}\right)_t$ = threshold bit weight per inch of bit diameter

This theoretical relation assumes perfect borehole cleaning and incomplete bit tooth penetration. Bingham (1965) suggested the following drilling equation based on considerable laboratory and field data. The equation can be written as(Bingham, 1965b):

$$ROP = K\left(\frac{W}{d_b}\right)^{a_5} N \qquad (3)$$

Here,

K = constant of proportionality that includes the effect of rock strength

a5 = bit weight exponent

In this equation, the threshold bit weight was assumed to be negligible and the bit weight exponent must be determined experimentally for the prevailing conditions.

iii) Weight on Bit: The significance of WOB can be shown as explained in Figure 6. The figure shows that no significant penetration rate is obtained until the threshold bit weight (Wt) is applied (Segment oa, i.e. up to point a). The penetration rate

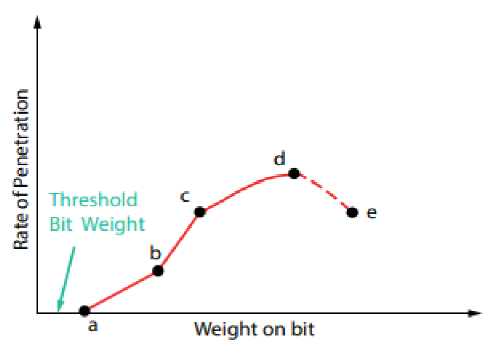

Fig 6 Typical bit weight response with ROP.

then increases rapidly with increasing values of bit weight (Segment ab). Then a constant rate of increase (linear) in ROP is observed at moderate bit weight (Segment bc). Beyond this point (c), only a slight improvement in the ROP (segment cd) is observed. In some cases, a decrease in penetration rate is observed at extremely high values of bit weight (Segment de). This behavior is called bit floundering. This is due to less efficient bottom-hole cleaning (because the rate of cutting generation has increased)(Mitchell and Miska, 2011; Hossain and Al-Majed, 2015).

2.1.6 Hydraulic Factor

The hydraulic factor is also called bit hydraulics. The two hydraulic factors affect the ROP greatly – i) jet velocity, and ii) bottom-hole cleaning. The mechanical factors of weight on the bit and rotary speed are then linearly related to the drilling rate, provided the hydraulic factors are in proper balance to ensure proper cleaning of the hole. The hydraulic factors affect the drilling rate only when they influence the rate of penetration or the efficiency of the drilling.

i) Jet Velocity: Significant improvements in penetration rate could be achieved by a proper jetting action at the bit. The improved jetting action promoted better cleaning of the bit face as well as the hole bottom. There exists an uncertainty on the selection of the best proper hydraulic objective function to be used in characterizing the effect of hydraulics on penetration rate. Bit hydraulic horsepower, jet impact force, Reynolds number, etc, are commonly used objective functions for describing the influence of bit hydraulics on ROP. Eckel (1968) studied microbits in a laboratory drilling machine. He made the most extensive laboratory study to date of the relation between penetration rate and the level of hydraulics. His study was at constant bit weight and rotary speed. Eckel proposed the following model based on the Reynolds number(Eckel, 1968):

$$N_{R_e} = K_s \frac{\rho_f \, v \, d_{nz}}{\mu_a} \qquad (4)$$

Here,

K_s = a scaling constant

ρ_f = drilling fluid density

v = flow rate

d_{nz} = nozzle diameter

μ_a = apparent viscosity of drilling fluid at 10,000 s–1

In Eq. (4), the shear rate of 10000 s –1 was chosen as representative of the shear rates present in the bit nozzle. The scaling constant, Ks, is somewhat arbitrary, but a constant value of 1/1,976 was used by Eckel to yield a convenient range of the Reynolds-number group. Figure 7 shows the experimental results of Eckel's finding. It is noted that increasing the Reynolds number function for the full range of the Reynolds number studied increased ROP. He found that when the bit weight was increased, the correlation curve simply shifted upward as shown in the figure(Eckel, 1968).

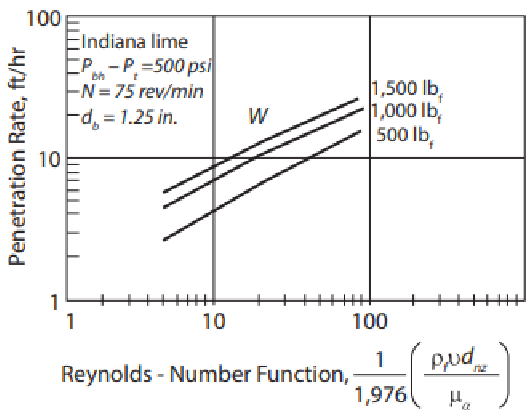

Fig 7 Effect of Reynolds number and WOB on ROP (Eckel, 1968). Penetration Rate, ft/hr

ii) Bottomhole Cleaning: It is one of the most important mechanisms for cutting transport in rotary drilling. However, proper bottom-hole cleaning is very difficult to achieve in practice. The jetting action of the mud crossing through the bit nozzles has to provide sufficient velocity and cross flow across the rock face to effectively remove cuttings from around the bit as the rock is newly penetrated. This would prevent cuttings from building up around the bit and teeth (bit balling), prevent excessive grinding of the cuttings clear them on their way up the annulus, and maximize the drilling efficiency. Many variables play a part in the efficiency of bottomhole cleaning. These variables include bit weight and rotation speed, bit type, flow rate, jet velocity, differential pressure, nozzle size, location and distance from the rock face, solids volume, cutting characteristics, etc. Proper bottom hole cleaning will eliminate excessive regrinding of drilled solids and will result in improved penetration rates. The efficiency of bottom hole cleaning can be achieved through the proper selection of bit nozzle sizes. The maximum hydraulic horsepower and the maximum impact force are the two requirements to get the best hydraulic cleaning at the bit. Both these items increase when the circulation rate increases. However, when the circulation rate increases, so does the frictional pressure drop. Inadequate hole cleaning can lead to costly drilling problems, such as mechanical pipe sticking, premature bit wear, slow drilling, formation fracturing, excessive torque and drag on the drill string, difficulties in logging and cementing, and difficulties in casings landing. The most prevalent problem is excessive torque and drag, which often leads to the inability to reach the target in high-angle/extended-reach drilling.

2.1.7 Drilling Fluid Properties

There are functions of drilling fluids that can have unique challenging influences. For example, the two mud properties that have a direct impact on hole cleaning are viscosity and density. The main functions of density are mechanical borehole stabilization and the prevention of formation-fluid intrusion into the annulus. Any unnecessary increase in mud density beyond fulfilling these functions will have an adverse effect on the ROP. This density increase may cause fracturing of the formation under the given in-situ stresses. So, mud density should not be used as a criterion to enhance hole cleaning. In contrast, viscosity has the primary function of the suspension of added desired weighting materials, such as barite. The following drilling fluid properties affect the ROP – i) density (i.e. mud weight), ii) rheological flow properties (i.e. viscosity), iii) filtration characteristics (i.e. filtrate loss), iv) solid content and size distribution, and v) chemical composition.

i) Density: An increase in drilling fluid density causes a decrease in the penetration rate for the rolling cutter bit. The density of the mud controls the pressure differential across the zone of crushed rock beneath the bit. If the density increases, it causes an increase in the bottom hole pressure beneath the bit. Thus, there is an increase in the pressure differential between the borehole pressure and the formation fluid pressure. Cunningham and Eenink (1959) conducted experiments on Berea sandstone and clay/water mud. Figure 8 shows the effect of mud density (i.e. mud weight) on ROP based on the experimental data. It shows a wider range of borehole and formation fluid pressure. Note that a good correlation is obtained when the data are replotted with the drilling rate as a function of overbalance (right side of Figure 8)(Cunningham and Eenink, 1959). Paiaman et al. (2009) reported that ROP decreases with the increase in mud weight as shown in Figure 9(Paiaman *et al.*, 2009).

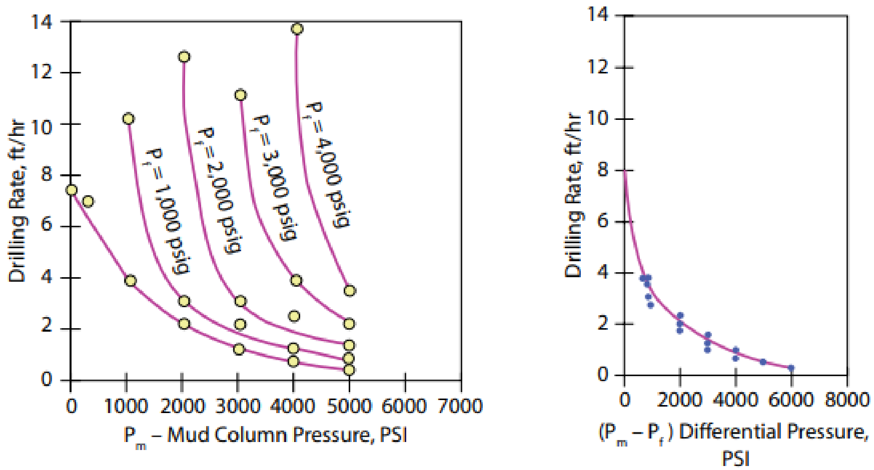

Fig 8 Effect of Mud Weight on Rate of Penetration (Bourgoyne, 1986).

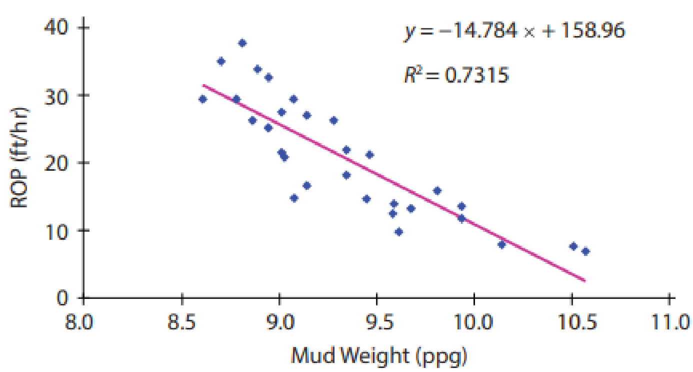

Fig 9 Effect of Mud Weight on Penetration Rate (Paiaman et al., 2009).

Effect of Mud Density (Overbalance) on Penetration Rate:

Bourgoyne and Young (1974) observed that the relation between overpressure and penetration rate could be represented approximately by a straight line on semi-log paper for the range of overbalance commonly used in field practice. In addition, they suggested normalizing the penetration rate data by dividing by the penetration rate corresponding to zero overbalance (borehole pressure equal to formation pressure). Figure 10 shows the normalized ROP and overbalance for the data as suggested by Bourgoyne and Young. Note that a reasonably accurate straight-line representation of the data is possible for moderate values of overbalance(Bourgoyne Jr and Young Jr, 1974b).

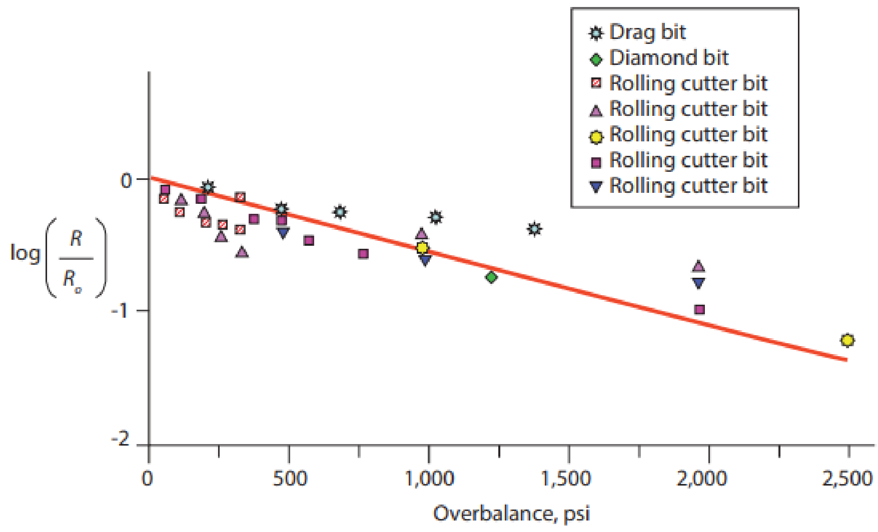

Fig 10 Exponential relation between penetration rate and overbalance for roller-cone bits (Mitchell and Miska, 2011).

ii) Viscosity: Penetration rates tend to decrease with increasing viscosity (Figure 11)(Alum and Egbon, 2011). The fluid viscosity controls the parasitic frictional losses in the drill string and thus the hydraulic energy available at the bit jets for cleaning. There is also experimental evidence that increasing plastic viscosity reduces penetration rates even when the bit is perfectly clean (Figure 12). Plastic viscosity is part of the resistance to flow caused by mechanical friction. The friction is caused by the viscosity of the fluid phase of the drilling mud. In addition, viscosity acts on the mobility of cuttings. With a high viscosity, cuttings tend to remain stuck on the bottom involving their re-drilling and thus a reduction in the performance of the bit. The best rates of penetration will be obtained with a fluid having a viscosity as low as possible at the exit of the nozzles of the bit (Figure 12)(Paiaman *et al.*, 2009).

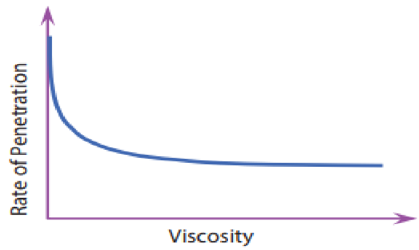

Fig 11 ROP vs. viscosity plot(Alum and Egbon, 2011).

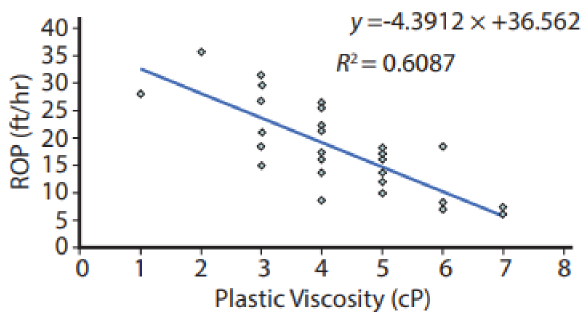

Fig 12 ROP vs. plastic viscosity plot(Paiaman *et al.*, 2009).

iii) Filtration Characteristics: Penetration rate increases with increasing filtration rate. The filtration characteristics of the mud control the pressure differential across the bit nozzle and formation rock.

iv) Solid Content and Size Distribution: Drilling muds are usually composed of a continuous fluid phase in which solids are dispersed. Plastic viscosity is part of the resistance to flow caused by mechanical friction. The friction is caused by solid concentration, size and shape of solids, and viscosity of the fluid phase. The penetration rate decreases with increasing solids content (Figures 13a and 13b). Figure 13a shows the variation of solid content rate of penetration and on the other hand, Figure 13b shows the effect of solid content on ROP at constant plastic viscosity. The solids content also controls the pressure differential across the bit nozzle and formation rock. For practical field applications, plastic viscosity is regarded as a guide to solids control. Plastic viscosity increases if the volume percent remains constant, and the size of the particle decreases. Decreasing particle size increases surface area, which increases frictional drag.

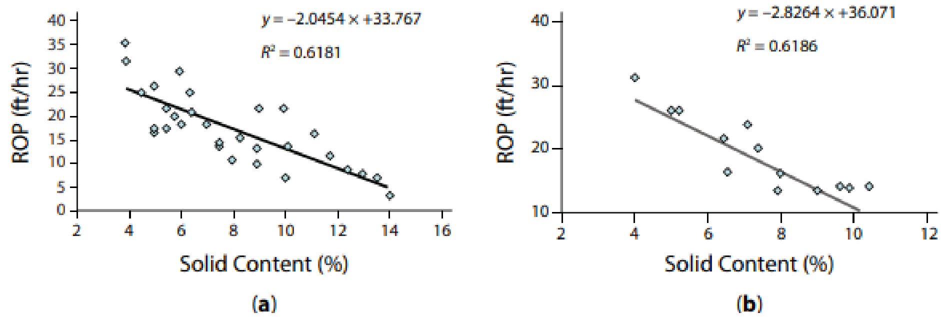

Fig 13 ROP vs. solid content plot (Paiaman et al., 2009).

v) Chemical Composition: The chemical composition of the fluid affects penetration rate, such that the hydration rate and bit-balling tendency of some clays are affected by the chemical composition of the fluid.

2.1.8 Bit Toot Wear

Most bits tend to drill slower as the drilling time elapses because of tooth wear. The tooth length of milled tooth rolling cutter bits is reduced continually by abrasion and chipping. The teeth are altered by a hard-facing or case-hardening process to promote a self-sharpening type of tooth wear. However, while this tends to keep the tooth pointed, it does not compensate for the reduced tooth length. The teeth of tungsten carbide insert-type rolling cutter bits and PDC bits fail by breaking rather than by abrasion. Often, the entire tooth is lost when breakage occurs. Reductions in penetration are rare due to bit wear and usually are not as severe for insert bits as for milled tooth bits unless a large number of teeth are broken during the bit run.

Several authors have published mathematical models for computing the effect of cutting-element wear on the penetration rate for roller-cone bits. Galle and Woods (1963) published the following model(Galle and Woods, 1963b):

$$ROP \propto \left(\frac{1}{0.928125h^2 + 6h + 1} \right)^{a_7} \qquad (5)$$

Here,

h = the fractional tooth height that has been worn away, in

a_7 = tooth wear exponent

A value of 0.5 was recommended for the exponent a7 for self-sharpening wear of milled tooth bits, the primary bit type discussed in Galle and Woods (1963). Bourgoyne and Young (1974) suggested a similar but less complex relationship(Bourgoyne Jr and Young Jr, 1974c):

$$ROP \propto exp\left(-a_7 h\right) \quad (6)$$

Bourgoyne and Young suggested that the exponent a_7 be determined based on the observed decline of penetration rate with tooth wear for previous bit runs under similar conditions.

2.2 History of Drilling Optimization

A historical timeline for drilling optimization is shown in Figure 7.41. One of the first attempts for the drilling optimization purpose was presented in the study of Graham and Muench in 1959. They analytically evaluated the weight on bit and rotary speed combinations to derive empirical mathematical expressions for bit life expectancy and drilling rate as a function of depth, rotary speed, and bit weight. Galle and Woods (1963) produced graphs and procedures for field applications to determine the best combination of drilling parameters. One of the most important drilling optimization studies was performed by Bourgoyne and Young (1974). They proposed the use of a linear drilling penetration rate model and performed multiple regression analyses to select the optimized drilling parameters. They used a minimum cost formula, showing that the maximum rate of penetration may coincide with the minimum cost approach if the technical limitations were ignored. In the mid-1980s operator companies developed techniques of drilling optimization in which their field personnel could perform optimization at the site referring to the graph templates and equations. In the 1990s different drilling planning approaches were brought to the surface (Bond et al., 1996, Carden et al., 2006). New techniques identified the best possible well-construction performances. Later on, "Drilling the Limit" optimization techniques were also introduced (Schreuder and Sharpe, 1999). Towards the end of the millennium real-time monitoring techniques started to take place, e.g. drilling parameters started to be monitored from off locations. A few years later real-time operations/support centers started to be constructed. Some operators proposed advanced techniques for monitoring drilling parameters at the rig site. Following the early developments in the rotary drilling system, groundbreaking developments in the latter years of the century took place. Highly inclined wells were drilled using rotary steerable; pressure-controlled drilling techniques with the acquisition of drilling parameters.

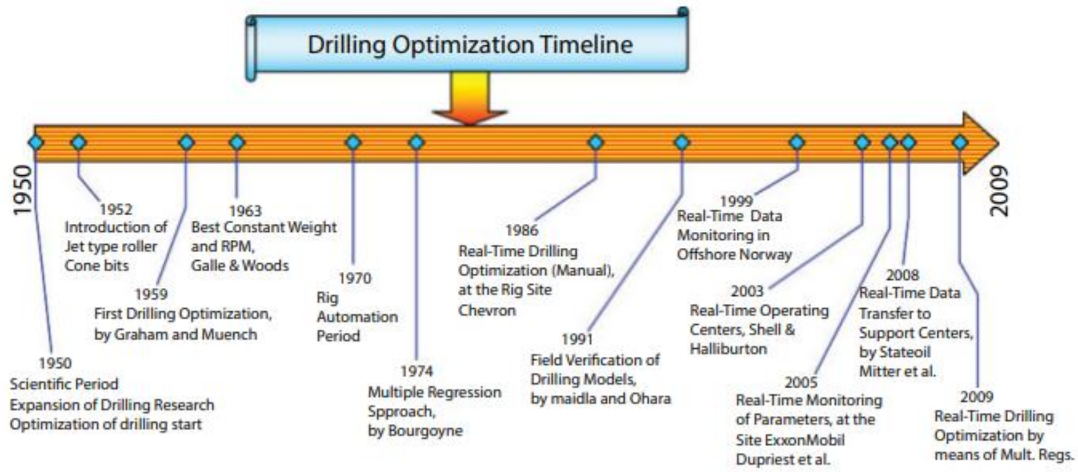

Fig. 14 Timeline for drilling optimization (Eren and Ozbayoglu, 2010).

2.3 Parameters for Drilling Optimization

Actual data is the only source of information to make a recommendation to optimize drilling operations. The parameters those of which could be collected from a drilling activity are as listed in Table 2. Each parameter to be collected from the rig site is going to have an impact on the overall optimization process. Data reliability and accuracy is very important, all of the data-collecting sensors should be accurately calibrated and signal the correct magnitude of measurement. The success of drilling optimization is closely related to the quality of the recorded drilling parameters. The parameters recorded for drilling optimization are critically important to be representative of the data they are meant to reflect. A brief description of the drilling parameters is deemed necessary to be explained.

Whenever the cost of drilling activity is reduced, the whole process is considered optimized. Optimization could be performed using adjusting the magnitude of two or more independent parameters. This could be achieved mainly using i) minimized cost per foot, and ii) minimizing problems. The cost of a drilling process could be minimized using working with an optimized combination of controllable drilling parameters. Hole problems those of which are being generated due to inefficient parameter usage generally occurring at the rig sites could be avoided. Drilling optimization considers that the rig equipment, BHA, and the bit to be used are already in the optimum selections. To achieve the objective of minimum cost drilling the bit should be prevented from damage when run into the hole.

WOB	Drill string Properties
RPM	Casing details
Pump parameters	Drilling fluid properties
Depth	Torque
Inclination	Hook-load
Azimuth	LWD
ROP	MWD

Table 2 Parameters from a drilling activity.

2.4 Factors Affecting the Drilling Operations

Some parameters significantly affect drilling operations. These parameters are normally used for drilling optimization. Therefore, it is important to know about those parameters. In general, drilling parameters may be broadly classified under two types – i) rig and bit-related parameters, and ii) formation parameters. The rig and bit-related parameters can be controlled but the formation parameters have to be dealt with. The formation parameters recorded for drilling optimization are critically important to be representative of the data they are meant to reflect. Many drilling parameters affect the performance of the drilling operation. If they are not adjusted properly, they will make the operation less economical. Rig and bit type parameters are broadly categorized as weight on bit or hook load, rotational speed (RPM), torque, and hydraulic parameters (i.e. Bit hydraulics) – flow rates, density of drilling fluid, etc. However, WOB, RPM, flow rate, bit hydraulics, and more importantly the type of bit are the most important drilling parameters affecting drilling operations because they are affecting the rate of penetration (drilling speed) and the economics of drilling. The parameters that come under the formation type are local stresses, mineralogy, formation fluids, rock compaction, and abrasive formation. Beyond the above-stated parameters, determining the rate of penetration is among the most sought aft er parameters in the drilling industry. This is because it allows for optimization of drilling parameters to decrease drilling costs and enhance drilling process safety. Among the above factors, some of the parameters are discussed below.

Weight on Bit: Represents the amount of weight applied to the bit. It is the abbreviation for "Weight on Bit". This load is then transferred to the formation which in turn is the energy created together with string speed that advances the drill string. It is measured through the drilling line, usually using having attached a strain gauge, which measures the magnitude of the tension in the line itself, and gives the weight reading based on the calibration. This sensor measures a unique value, which is the overall weight (Hook-load) of the string including the weight of the block and Top Drive System (TDS). For all of these circumstances, correct calibration is required to have a proper reading for this drilling parameter. Field study shows through testing that doubling the bit RPM in 6,000-psi rock while keeping WOB constant resulted in a 70% increase in ROP. However, doubling WOB, with RPM constant, resulted in a 300% increase in ROP. Bit condition is very important as there is blunting while drilling progresses, which depends on the formation being drilled.

RPM: This parameter stands for "revolutions per minute". It represents the rotational speed of the drill string. With the invention of TDS, the reading is directly linked to the electronics of the unit itself. It is considered that the measurements for this parameter are accurate as long as the acquisition system set-up has been thoroughly made up.

Torque: This parameter is the torque of the drill string while it is rotating. It is measured using TDS systems. Previously the readings for this parameter were relative. This parameter is going to be significantly important for inclined and highly deviated wellbores, which is also related to the wellbore cleaning issues.

Pump Parameters: The pump parameters are composed of the liner size in use, pump strokes, and the pump pressure. In case two pumps are working simultaneously all of the data for two of the pumps should be acquired. With the electric pumps, the stroke is transmitted in the same way as RPM. The pressure at the pump in case of having been acquired could be compared with the reliability of the standpipe pressure. Pump pressure should always be greater than the standpipe pressure. The use of flow meters could also be adapted for accurate flow rate measurements.

Depth: The value of depth, in other, means the bit position is input in the mudlogging unit (MLU). The operator is responsible for that. Usually, it is linked to the position of the block, using the sensors located at the crown block.

Inclination – Azimuth: These two parameters are the responsibility of the directional driller. Efficient communication between the MLU and the measurement while drilling (MWD) unit is to the benefit of these two parameters, which may be very important for wellbore stability considerations.

ROP: This parameter is the most important since all of the calculations in this study are based on estimations of ROP. It is measured through the relative change of the position of the block in time. Accurate calibrations are very important to have a representative ROP parameter.

String–Casing Properties: The string and casing properties are very important when the frictional pressure losses are to be calculated.

Fluid Properties: Rheological properties and the density of the drilling fluids are also among the very important parameters to be recorded for optimization purposes. Usually, the drilling fluid density is measured through calibrated mud weight (MW) sensors. Rheological properties on the other hand are still measured manually. Recent developments regarding real-time pipe viscometers dictate alternative solutions. There are experimental studies performed in the laboratory using pipe viscometers. Continuous real-time viscometer probes placed on the flow line could facilitate data acquisition over the rheological properties of drilling fluids in real-time.

Logging While Drilling (LWD): Formation-related parameters could be captured during drilling and used in the optimization process. LWD consideration needs to be applied to enhance the optimization process.

2.5 How to Optimize the Drilling Operations

The optimization of the drilling performance, even more in a field development context requires i) data acquisition and ii) data processing. Data acquisition has the relevant measurement of drilling data such as WOB, RPM, torque, ROP, and flow rate. Some of these data are time-dependent. Data processing is based upon relevant drilling data, the processing leads to drill bit response follow-up with mechanical specific energy (MSE), HMSE logs, an E/S Diagram, events identification (steady drilling, vibrations, cleaning issue, wear development, and drilling response optimization (drilling parameters adaptation). The surface data allows tracking of the drilling behavior. This is especially possible when the full-bit design is known. In other cases, detailed information on the design needs to be provided (cutter distribution, orientation, and position). Using the bit signature together with the GeoScan lithology, we can then assess on a real-time basis the drilling performance. The wear phenomena can be tracked and a wear log is created. The vibrations are also monitored using surface data, bit signature, and predicted ROP.

Two main services are provided in this regard i) real-time service (RTS), and ii) next well service (NWS). The RTS provides the decision maker with the information needed to modify and optimize the drilling parameters in order to increase ROP, bit life, and decrease vibrations. Pull-out decisions can also be supported by this RTS. NWS uses the parameters logs recorded during drilling operations and electric logs. The lithology can also be reviewed and the field mapped, well after well. Lessons learned are gathered. This leads to the construction of a reference knowledge database (RKD). BHA components and bit design can then be adapted to improve performance for the next well. Guidelines are given to the drillers.

Figure 15 summarizes the algorithmic steps, which should be followed consecutively to optimize the drilling parameters of a hydrocarbon field using mathematical solutions. As shown in this figure, some of the comparative optimization results such as the selection of optimal bit and mud properties will be employed in the numerical technique. Each step of this algorithm can be achieved by using the proper mathematical model for each parameter optimization.

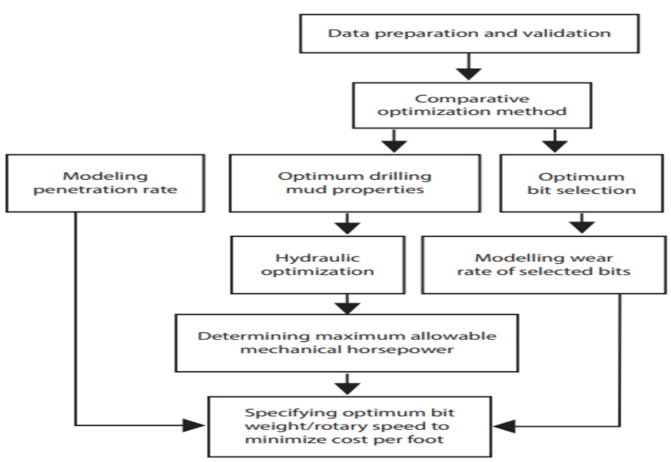

Fig. 15 Algorithm of the mathematical optimization procedure.

2.6 Traditional Optimization Process

The traditional optimization process consists of (i) pre-run modeling, (ii) real-time data measurement and monitoring, and (iii) post-run analyses and knowledge management. At the center of this process, the team members are the personnel who are experts in these technologies and who can make recommendations to avoid trouble and improve drilling performance. Generally, a comprehensive drilling optimization should include solutions for i) drill string integrity, ii) hydraulics management, and iii) wellbore integrity. However, new drilling optimization technologies emphasize information management and real-time decision-making. On the other hand, the traditional three-step optimization process will not fit the real-time process and has had to be changed. First pre-run modeling needs to be changed to "real-time modeling". This change is required because the input parameters for pre-run models have typically been outdated and incorrect. Therefore, modeling results were often of little use for real-time decision-making. Second, integrated real-time modeling and data are required to allow detailed diagnoses of the downhole environment. Third, a rig-to-office integration is best so the optimization process can be monitored 24/7 by an asset team. These three new technologies have been summarized by (Chen, 2004) as i) real-time modeling, ii) integrated real-time modeling and data, and iii) a real-time operation center (RTOC).

i) Real-Time Modeling: The objective of optimizing drilling parameters in real-time is to arrive at a methodology that considers – i) past drilling data, and ii) predicts drilling trends advising optimum drilling parameters. Real-time optimization is needed to save drilling costs and reduce the probability of encountering problems. Conventional modeling is usually run during well-planning to avoid a set of predicted data. As drilling processes, the input parameters may change intentionally or unintentionally. As a result, conventional, stand-alone computer software requires constant manual updating to produce pertinent results. Such a procedure, however, has proven to be impractical. In contrast, real-time modeling is automatically updated using "correct" input data, which is no doubt more accurate. In addition, real-time modeling is always "on" allowing continuous monitoring to prevent drilling accidents. Real-time modeling also allows integration with real-time data to enable real-time decision-making. To date, several real-time drilling optimization-related modeling programs are being developed as BHA dynamics, torque and drag, pore pressure/fracture gradient prediction, hydraulics, hole cleaning, and wellbore stability.

ii) Integrated Real-Time Modeling and Data: Although real-time modeling produces better results than conventional, stand-alone modeling, the delivery of useful information in a useful form and diagnosis of a problem requires the integration of modeling with downhole data. For example, the integration of the following models and data is always beneficial:

• Bottomhole Assembly (BHA) dynamics model with downhole vibration data

• Pore pressure model with Pressure While Drilling (PWD) and Formation Testing While Drilling (FTWD) data

• Hydraulic model with PWD data

• Hole cleaning model with PWD and solids in mud

• Wellbore stability model with Logging While Drilling (LWD) imaging data

iii) RTOC: The first Real-Time Operation Center (RTOC) was set up by Shell E&P in New Orleans in early 2002. Since then, several other RTOCs for different operators have been developed particularly for offshore rigs. There are many reasons to set up RTOCs. First, wells drilled offshore are very expensive. They require full attention from the best staff available. Second, critical decisions are always multidisciplinary; and multidisciplinary decision-making with expert staff is impractical to arrange at a rig. Third, a permanent, common ground needs to be identified for office and off-shore staff throughout planning and execution; and RTOCs readily satisfy this element. Lastly, full-time (24/7) real-time drilling optimization monitoring and information management is required to avoid hazards; and 24/7 monitoring available to key personnel is best done by an RTOC.

3. Literature Review

There are many studies about modeling and optimization of the rate of penetration (ROP):

Graham and Muench, in 1959 Executed the initial drilling optimization attempt by quantitatively assessing the WOB and RPM combinations to produce empirical mathematical formulas for bit life expectancy.

Maurer, in 1962 Developed a theoretical relationship between ROP and bit weight, rotary speed, bit size, and rock strength for rolling cutter bits.

Galle and Woods, in 1963 Created graphics and procedures for field applications to discover the perfect drilling parameter combination. Also, they studied the influence of bit wear on ROP for roller-cone bits.

Bingham, in 1965 Suggested a drilling equation of ROP as a function of RPM based on laboratory and field data.

Bourgoyne and Young, in 1974 Suggested the use of multiple regression analysis using a linear drilling penetration rate.

Bourgoyne and Young, in 1974, and Paiaman, et al., in 2009 Studied the relationship between ROP and mud density.

Operator companies, in the mid-1980s, approached drilling optimization strategies so that their field staff could attempt optimization at the location utilizing the conditions and chart formats.

Reza and Alcocer, in 1986 Created a dynamic non-linear, multidimensional, dimensionless drilling model (ROP, bit dulling rate, and bearing wear rate) utilizing the Buckingham.

Onoe, et al. in 1991 Outlined the idea, capabilities, and development of a cutting-edge real-time information system for drilling that would significantly improve drilling efficiency and engineering correctness, as well as operational safety and data management.

Teale, in 1962, and Pessier and Fear, in 1992 Studied the mechanical specific energy (MSE) as a function of WOB and ROP.

Bond, et al.in 1996, and Carden, et al., in 2006 Presented various drilling planning strategies.

Schreuder and Sharpe, in 1999 Presented the leading well development exhibitions utilizing modern strategies and introducing optimization strategies for limit drilling.

Davis, in 2002 Analyzing data with multiple variables involves characterizing an observation unit to predict ROP.

Montgomery and Runger, in 2003 Studied the method of multiple regression models with several regressor variables to predict ROP.

Chen, in 2004 Summarized three new technologies i.e. real-time modeling, coordinates real-time modeling and information, and a real-time operation center.

Ozbayoglu, et al., in 2004 Performed a comprehensive sensitivity analysis on the drilling cuttings transport that controls ROP.

Thonhauser, in 2004 Examined performance while drilling and after drilling using process-related data acquired in real-time.

Maidla and William, in 2005 and 2010 Demonstrated how MSE was applied in a real-time drilling information system on the rig and at remote monitoring stations. Presenting the measurement approaches that involved autonomous drilling operations detections of common drilling activities and data quality control (QC).

Dupriest, et al., in 2005, Armenta, in 2008, and Khamis, in 2013 Analyzed MSE was used sparingly to look into particular field operations inefficiencies. Also, they studied the relationships between ROP and WOB.

Osgouei, in 2007 Constructing a drilling model to forecast and optimize the ROP by taking into account the effects of the various drilling parameters.

Rashidi, et al., in 2008 and 2010 (a & b) Introduced a novel technique to compute real-time bit wear by combining the MSE and ROP models and presenting the real-time use of a model created for bit wear analysis. Researching to show how altering drilling parameters—bit wear and design— affected ROP for both procedures.

Armenta, in 2008 Combined experimental and field data and established a unique connection to discover ineffective drilling settings.

Mohan, et al. in 2009 Addressed a novel correlation that uses MSE to discover inefficient drilling circumstances.

Vogel and Asker, in 2010 Displayed certain scenarios to educate administrators and other penetrating organizations about the cost-effectiveness and significance of genuine-time data management strategies and data exchange for complimenting innovation in drilling operations.

Staveley and Thow, in 2010 Showed techniques for enhancing teamwork and analyzing current and past drilling data, increasing the cost and drilling efforts' effectiveness.

Eren and Ozbayoglu, in 2010 Studied drilling optimization through the data transmission process and developed a drilling model to optimize their operations' parameters.

Sharma, et al., in 2010 Presented 6 case histories where the utilization of downhole boring information increments penetrating efficiency.

Mostofi, et al., in 2010 Generated a rock strength log of the Asmary formation using a drilling operation's backward simulation.

Eren and Ozbayoglu, in 2010, and Hossain and Al Majed, in 2015 Presented the timeline of the drilling optimization.

Alum and Egbon, in 2011 Studied the relationship between ROP and mud viscosity. Making a semi-analytical demonstration for ROP utilizing real-time bit recordings from wells penetrated within the Niger Delta supplies based on the first Bourgoyne and Young model.

Mitchell and Miska, in 2011 Presented a relationship between rock shear strength and threshold bit weight at 14.7 psi.

Zoellner, et al., in 2011 Examined several situations to track drilling hydraulics by comparing fluid flow to pump pressure and other pertinent sensor channels.

Gidh, et al. in 2011 Building a software solution based on an artificial neural network (ANN) to monitor the real-time ANN's application of operational parameters like WOB and RPM to boost overall ROP while optimizing bit life.

Koederitz and Johnson, in 2011 Discussed the creation and testing of an autonomous drilling system in the field.

Bataee and Mohseni, in 2011 Used ANNs to forecast the correct ROP, optimizing the drilling parameters, anticipating how long a well would take to drill, and ultimately lowering the cost of drilling future wells.

3.1 Established Models for Rate of Penetration

The main characteristic of rotary drilling penetration performance is not only the fracture of the rock on the bottom but also the removal of the fractured cuttings from the rock face in an instant and efficient manner to provide further fracturing and drilling progress. Due to the complexity of understanding the rate of penetration mechanism of drilling operations, industry pioneers have adopted empirical approaches by quantifying the effects of the controllable parameters on ROP performance, more than the analytical model implementation for the understanding of the rate of penetration in the industry of drilling. It is reportedly known that time spent drilling wells is composed of up to 30% "rotating time" of the total well construction time. Penetration rate optimization is consequently an important cost reduction consideration. It is assumed that all of the properties of the formations affecting the rate of penetration, that is subject to optimization are macroscopically homogeneous and with unique physical properties throughout the entire interval.

The development of an ROP model is a challenging job due to the known and unknown variables. As a result, numerous investigations have been done in this area. In considering which variables to choose for developing an ROP model, experience and research suggest the eight variables as mentioned earlier. These variables are mainly – i) mud properties, ii) hydraulics, iii) bit type, iv) weight on bit, v) drill string rotation speed, vi) depth, vii) bit tooth wear, and viii) formation properties. However, for horizontal and inclined wellbores, hole cleaning is also a major factor that influences the ROP. The basic interactive effects between these variables were determined by design experiments. Variable interaction exists when the simultaneous increase of two or more variables does not produce an additive effect as compared with the individual effects. The meaning of variable interaction is illustrated in Figure 16.

In addition, the mathematical model for the penetration rate could be written as a function of drilling parameters such as WOB/db, and RPM, as given in the below section. Also, the bit tooth wear has been considered in the same equation for optimization purposes.

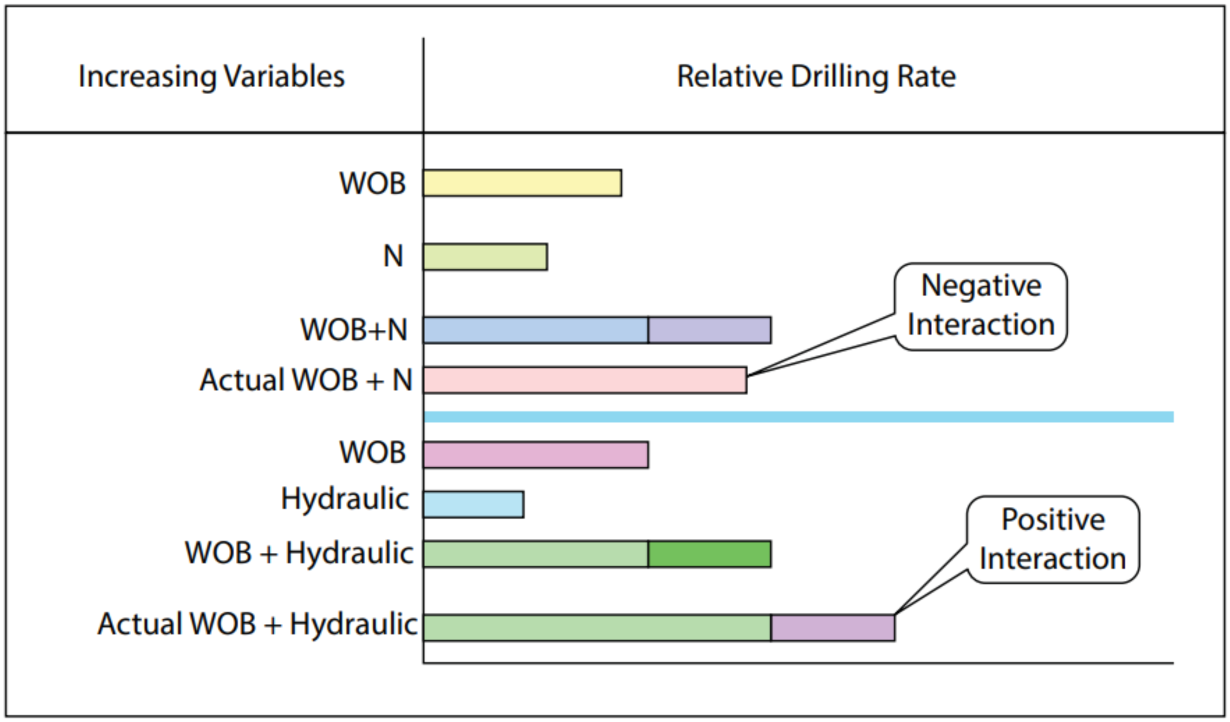

Fig. 16 Positive and negative interaction.

3.1.1 ROP Models

Graham and Muench (1959) are some of the first researchers who conducted evaluations on drilling data to determine the optimum weight on bit and rotary speed combination. They used a mathematical analysis method to drill under optimum circumstances for drilling-related costs. Empirical mathematical expressions were derived for bit life expectancy and the drilling rate as a function of depth, rotary speed, and bit weight. The proposed mathematical relations contained constants representative of the respective formations existing in the area. Their study resulted in being able to propose optimum weight on bit and rotary speed using calculations under any drilling conditions to minimize total drilling costs. There are three most widely used models for estimating the rate of penetration; i) Maurer, ii) Galle and Woods, and iii) Bourgoyne and Young.

3.1.1.1 Maurer's Method

Maurer (1962) derived the ROP equation for roller-cone type of bits considering the rock cratering mechanisms. His method was developed based on a theoretical penetration equation as a function of WOB, RPM, bit size, and rock strength. The developed equation was based on observations such as the amount the crater cutter can create, and rock strength-related considerations. In addition, it was based on a 'perfect cleaning' condition where all of the rock debris is considered to be removed between tooth impacts. A working relation between drilling rate, weight on bit, and string speed was achieved. It was also mentioned that the obtained relationships were a function of the drilling depth.

Maurer rate of drilling equation is expressed as:

$$\frac{dF_D}{dt} = \frac{4}{\pi d_b^2}\frac{dV}{dt} \qquad (7)$$

Here

F_D = distance drilled by bit, ft.

t = time, hr

V = volume of rock removed,

d_b = bit diameter

3.1.1.2 Galle and Woods' Method

Galle and Woods (1963) were some of the first researchers to investigate the effect of best constant bit weight and rotary speed for the lowest cost and developed semi-empirical equations. They investigated the effects of weight on bit, rotary speed, and cutting structure dullness on drilling rate, rate of tooth wear, and bearing life. They presented graphs and procedures for field applications to determine the best combinations of constant weight and rotary speed. They assumed a relation for the wear rate as a function of time in relation to the inverse ratio of bit weight to bit diameter. The given equation was limited with a load application of 10,000 lbf /in of bit diameter. They also published an equation showing a relationship between the tooth wear rate and the rotary speed for only milled tooth bits designed for soft formations. In their graphs, the drilling cost, footage, drilling hours, and condition of teeth and bearing of the dull bit may be calculated. The drilling costs were demonstrated to be reduced using the recommended combinations of the drilling parameters.

They presented the drilling rate equation as given in (8) as a function of WOB and RPM.

$$\frac{dF_D}{dt} = C_{fd} \frac{\bar{W}^k}{a^p} r \qquad (8)$$

Here

C_{fd} = formation drill ability parameter

a = 0.028125h2 + 6.0h + 1

h = bit tooth dullness, fractional tooth height worn away, in

p = 0.5 (for self-sharpening or chipping type bit tooth wear)

k = 1.0 (for most formations except very soft formations), 0.6 (for very soft formations)

r = function of N which can be expressed as Eq. (9) and Eq. (10)

$\bar{W}$ = function of WOB and d_b, such that $\bar{W} = \frac{7.88\,WOB}{d_b}$

Now, r can be expressed for two types of formations.

For hard formation:

$$r = e^{\frac{-100}{N^2}} N^{0.428} + 0.2N\left(1 - e^{\frac{-100}{N^2}}\right) \qquad (9)$$

For soft formation:

$$r = e^{\frac{-100}{N^2}} N^{0.750} + 0.5N\left(1 - e^{\frac{-100}{N^2}}\right) \quad (10)$$

Here

N = rotational speed

Galle and Woods (1963) also defined the rate of dulling and bearing life equation respectively as shown in Eq. (11) and Eq. (12).

Rate of the dulling equation:

$$\frac{dh}{dt} = \frac{1}{A_f}\frac{i}{am} \quad (11)$$

where:

$$i = N + 4.348\times10^{-5} N^3$$
$$m = 1359.1 - 714.19\log_{10}\overline{W}$$

Bearing life equation:

$$B = S\frac{L}{N} \quad (12)$$

where:

S = drilling fluid parameter

L = tabulated function of $\overline{W}$ used in bearing life equation

3.1.1.3 Bingham Model

Bingham (1965) proposed a rate of penetration equation based on laboratory data as stated in Eq. (13). In his equation, the threshold bit weight was assumed to be negligible and the rate of penetration was a function of the applied weight on the bit and the rotary speed of the string. The bit weight exponent, a5 was set to be determined experimentally through the prevailing conditions.

$$ROP = K\left(\frac{WOB}{d_b}\right)^{a_5} N \quad (13)$$

Here

ROP = rate of penetration

K = proportionality constant for rock strength effect

a_5 = bit weight exponent

3.1.1.4 Bourgoyne and Young's Method

Bourgoyne and Young's (1973 and 1974) method is the most important drilling optimization method since it is based on a statistical synthesis of past drilling parameters. A linear penetration model is being introduced and multiple regression analysis over the introduced rate of penetration equation is being conducted. For that reason, this method is considered to be the most suitable method for real-time drilling optimization. They developed a mathematical model and a summary of the equations is given below.

The rate of penetration is expressed as:

$$\frac{d}{dt}(ROP) = e^{\left(a_1 + \sum_{i=2}^{8} a_i x_i\right)} \quad (14)$$

Here a1 = formation strength parameter

i = index number for ith drilling rate of penetration equation coefficient or summation index for ith data point

ai = set of constants that relates to each of the drilling parameters considered

xi = set of dimensionless drilling parameters calculated from the actual collected drilling data

The normalization constants given in the general ROP Eq. (14) are modified accordingly as a function of the data property when used as an input to the regression cycle. When modified normalization constants are used, the coefficients should give accurate predictions for ROP. So, the dimensionless drilling parameters in Eq. (14) are described as follows:

Formation Resistance:

$$x_1 = 1.0 \quad (15)$$

Consolidation Effects:

$$x_2 = 10,000 - TVD \quad (16)$$

Overpressure Effects:

$$x_3 = TVD^{0.69}\left(g_p - 9.0\right) \quad (17)$$

Differential Pressure:

$$x_4 = TVD\left(g_p - \rho_{ec}\right) \quad (18)$$

Bit diameter and WOB:

$$x_5 = \ln\left\{\frac{\frac{WOB}{d_b} - \left(\frac{WOB}{d_b}\right)_t}{4.0 - \left(\frac{WOB}{d_b}\right)_t}\right\} \quad (19)$$

Rotary Speed:

$$x_6 = \ln\left\{\frac{N}{100}\right\} \quad (20)$$

Tooth Wear:

$$x_7 = -h \quad (21)$$

Bit Hydraulic:

$$x_8 = \ln\left\{\frac{\rho_m Q}{350\,\mu\, d_n}\right\} \quad (22)$$

Here

TVD = total vertical depth, ft.

g_p = pore pressure gradient of the formation, lbf /gal

ρ_{ec} = equivalent circulating mud density at the bottom hole, lbf /gal

$\frac{WOB}{d_b}$ = weight on bit per inch of bit diameter, 1000 lbf /in

$\left(\frac{WOB}{d_b}\right)_t$ = threshold weight on bit per inch of bit diameter, 1000 lbf /in

h = bit tooth dullness, a fraction of original tooth height worn away

ρ_m = mud density, lbf /gal

Q = flow rate, gal/min

μ = Viscosity

d_n = bit nozzle diameter, in

The constants given in Eq. (14), a1 through a8 should be determined through the multiple regression analysis using the drilling data. They represent the effects of formation strength [Eq. (15)], compaction effect [Eq. (16)], overpressure [Eq. (17)], pressure differential [Eq. (18)], bit weight [Eq. (19)], rotary speed [Eq. (20)], tooth wear [Eq. (21)] and hydraulic exponent [Eq. (22)].

The threshold weight on bit and bit diameter value is not a constant, significantly, it may have varying magnitudes based on formation characteristics, and for this reason whole data trend is observed when this threshold value is determined as an input. The same value could easily be obtained from a drill-off test. The fractional tooth height calculation methodology is a function of reference abrasiveness constants in the same field and is related to the time the bit in use has operated. Therefore, combining the Eqs. (14-22), the open form of the general ROP Eq. (23) for roller cone bit types is given as:

$$\frac{d}{dt}(ROP) = e^{\left[a_1 + a_2(10000 - TVD) + a_3 TVD^{0.69}(g_p - 9.0) + a_4\, TVD(g_p - \rho_{ec}) + a_5 \ln\left\{ \frac{\frac{WOB}{d_b} - \left(\frac{WOB}{d_b}\right)_t}{4.0 - \left(\frac{WOB}{d_b}\right)_t} \right\} + a_6 \ln\left\{\frac{N}{100}\right\} + a_7(-h) + a_8 \ln\left(\frac{\rho_m Q}{350\, \mu\, d_n}\right) \right]} \tag{23}$$

Here

al = formation strength parameter

a2 = exponent of the normal compaction trend

a3 = under compaction exponent

a4 = pressure differential exponent

a5 = bit weight exponent

a6 = rotary speed exponent

a7 = tooth wear exponent

a8 = hydraulic exponent

The considered effects of the controllable and uncontrollable drilling variables on ROP are individually described below for each item. Figure 17 gives the schematically represented general rate of penetration equation for roller-cone bit types.

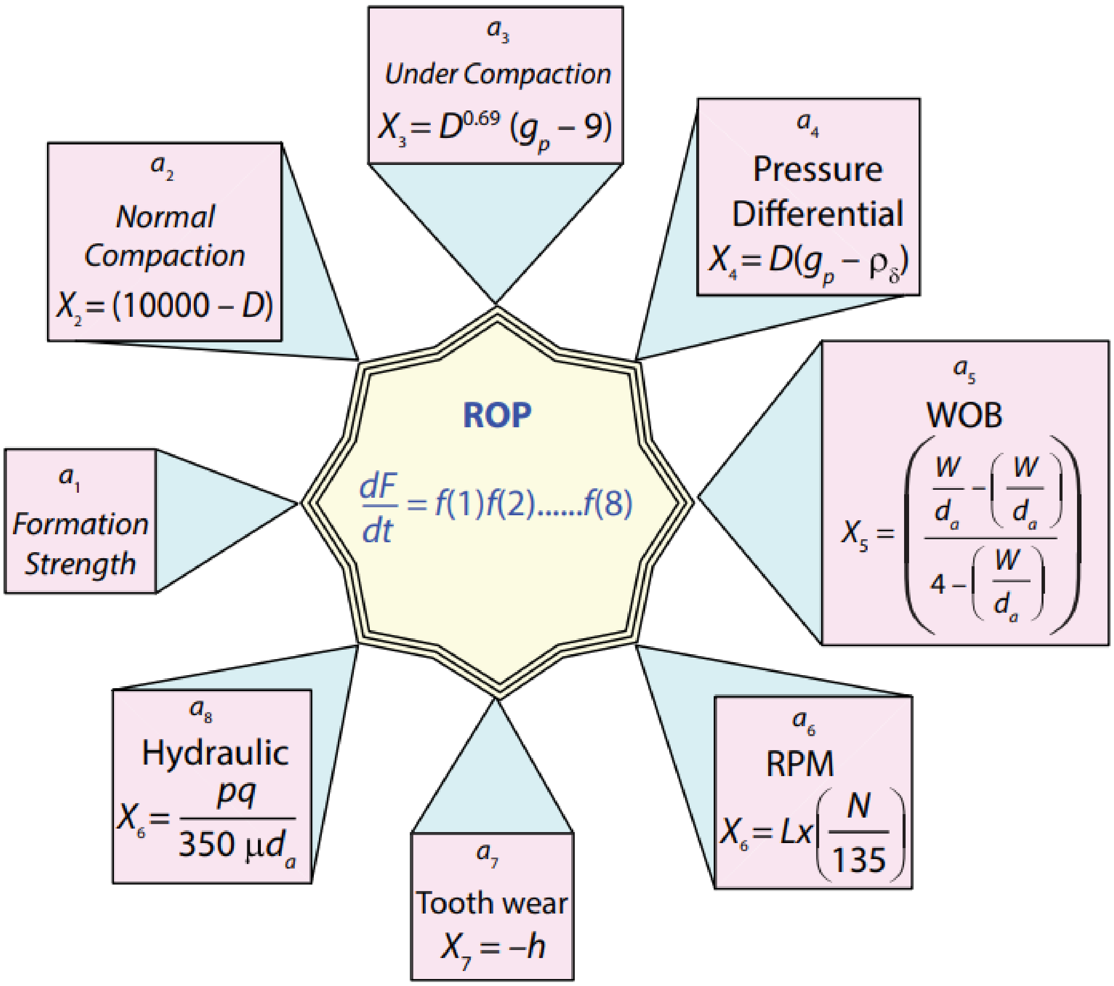

Fig. 17 General rate of penetration equation.

3.1.1.5 Reza and Alcocer Method

Reza and Alcocer (1986a) developed a dynamic non-linear, multidimensional, dimensionless drilling model for deep drilling applications using the Buckingham π - theorem. Buckingham π - theorem is a dimensional analysis theorem used to generate equations in dimensionless forms. The model is based on three equations – rate of penetration, rate of bit dulling, and rate of bearing wear. Their study reflected the effects of the following variables on three given equations: weight on bit, rotary speed, bit diameter, bit nozzle diameter, bit bearing diameter, drilling fluid characteristics (density and viscosity), drilling fluid circulation rate, differential pressure, rock hardness, temperature, and heat transfer coefficient. They defined the rate of penetration as given in Eq. (25) in the form of a non-linear, multivariable equation.

$$\frac{ROP}{N\,d_{bd}} = C_1\left[\frac{Nd_{bd}^2}{\upsilon}\right]^a\left[\frac{Nd_{bd}^3}{Q}\right]^b\left[\frac{Ed_{bd}}{WOB}\right]^c\left[\frac{\Delta p\,d_{bd}}{WOB}\right]^d \quad (25)$$

Here

ROP = rate of penetration, ft ./min

C_1 = proportionality constant in penetration rate equation bd

d_{bd} = bearing diameter, in

υ = drilling fl uid kinematic viscosity, cp

Q = volumetric flow rate, gpm

E = rock hardness, psi

In Eq. (25), C1, a, b, c, and d are unknown parameters. To find the coefficients using the available data a linear regression analysis methodology was applied after taking the natural logarithm of both sides of the equation above. When the solution of the ROP equation was written the following relation was reported to investigate the deep well drilling problems, equation (26).

$$\frac{ROP}{N\,d_{bd}} = 0.33\left[\frac{Nd_{bd}^2}{\upsilon}\right]^{0.43}\left[\frac{Nd_{bd}^3}{Q}\right]^{-0.68}\left[\frac{Ed_{bd}}{WOB}\right]^{-0.91}\left[\frac{\Delta pd_{bd}}{WOB}\right]^{-0.15} \quad (26)$$

The general equation for the rate of bit dulling was obtained as in equation (27).

$$\frac{d_{bt}}{N\,d_b} = 0.001\left[\frac{Q}{N\,d_b^3}\right]^{0.56}\left[\frac{WOB}{E\,d_b^2}\right]^{0.26}\left[\frac{d_b}{Q}\right]^{-0.03} \quad (27)$$

Here

d_{bt} = bit tooth dullness, a fraction of the original tooth

d_b = bit diameter

The general equation for the bit-bearing life was obtained as in equation (28).

$$\frac{B_{bw}}{N} = 0.05\left[\frac{T\,h_t\,d_{bd}}{N\,WOB}\right]^{0.51}\left[\frac{\upsilon}{N\,d_{bd}^2}\right]^{0.4}\left[\frac{Q}{N\,d_{bd}^3}\right]^{-0.5} \quad (28)$$

Here

B_{bw} = bearing wear fraction of the total life

T = temperature at the bottom of the hole, °F

h_t = heat transfer coefficient, $BTU/{}^\circ F - ft^2 hr$

d_{bd} = bearing diameter, in

In the second part of their study, Reza and Alcocer (1986b) mentioned that the exponents of the derived models are sensitive to unknowns, and there would be fluctuations from region to region and well to well. For that particular reason, their finding was a generalization of the model specifically for a region and in deep fields/wells.

3.1.1.6 Warren's Model

Warren derived a model of the drilling process for tri-con bits called the perfect-cleaning model in 1987 and later modified by Hareland (Hareland and Hoberock, 1993). The basic idea is that under steady-state drilling conditions, the rate of cutting removal from the bit is equal to the rate at which new chips are formed. This implies that the cutting-generation process, the cutting removal, or a combination of the two processes controls the ROP. The perfect-cleaning model, which is shown in the following equation, is reviewed as a starting point for the development of an imperfect cleaning model.

$$ROP = \left[\frac{a\,S^2\,d_b^2}{N^b WOB^2} + \frac{c}{N\,d_b}\right]^{-1} \quad (30)$$

Here

a, b, c = bit constant for Warren's constant

S = confined rock strength, psi

Unfortunately, ROP in most field cases is significantly inhibited by the rate of cutting removal from under the bit. Thus Eq. (30) is not effective for predicting field ROP without modification to account for imperfect cleaning. Therefore, it is necessary to modify the ROP model for imperfect cleaning conditions, which happen because of real situations. Thus, the resultant expression for ROP is:

$$ROP = \left[\frac{a S^2 d_b^2}{N\, WOB^2} + \frac{b}{N\, d_b} + \frac{c\, d_b\, \gamma_f\, \mu}{F_{jm}} \right]^{-1} \quad (31)$$

Here

γ_f = fluid specific gravity

μ = mud plastic viscosity, cp

F_{jm} = modified jet impact force, klb_f

The modified impact force is calculated from the following equation:

$$F_{jm} = \left[1 - A_v^{-0.122} \right] F_j \quad (32)$$

Here

A_v = ratio of jet velocity to return velocity

F_j = jet impact force, klb_f

If A_v is the ratio of the jet velocity to the fluid return velocity, the A_v (for three jets) is given by:

$$A_v = \frac{v_n}{v_f} = \frac{0.15\, d_b^2}{3\, d_n^2} \quad (33)$$

3.1.1.7 Modified Warren's Model

Neither Winters (Winters et al., 1987) nor Warren (Warren, 1987) addresses Chip's hold-down effects on penetration rate modeling, but it is known that this effect is important. It is an estimation of the resultant forces on a chip when the bit generates it. To establish the best relationship for chip hold down, data from laboratory full-scale drilling tests was used in which bottom-hole pressure varied and other conditions remained constant. A reasonable fit to these different lithologies is given by:

$$f_c\left(P_e\right) = c_c + a_c\left(P_e - 120\right)^{b_c} \quad (34)$$

Here

P_e = differential pressure

$f_c\left(P_e\right)$ = chip hold down function

a_c, b_c, c_c = lithology-dependent constant

Units of a_c, b_c, c_c are chosen such that $f_c\left(P_e\right)$ is dimensionless. Equation (30) can now be modified to include chip hold down effect and becomes:

$$ROP = \left[f_c\left(P_e\right)\left(\frac{a\,S^2\,d_b^2}{N^b\,WOB^2} + \frac{b}{N\,d_b} \right) + \frac{c\,d_b\,\gamma_f\,\mu}{F_{jm}} \right]^{-1} \quad (35)$$

Hareland (Hareland et al., 1993) modified this ROP model for the effect of bit wear on ROP by introducing a wear function, W_f into the model:

$$ROP = W_f \left[f_c\left(P_e\right)\left(\frac{a\,S^2\,d_b^2}{N^b\,WOB^2} + \frac{b}{N\,d_b} \right) + \frac{c\,d_b\,\gamma_f\,\mu}{F_{jm}} \right]^{-1} \quad (36)$$

$$W_f = 1 - \frac{\Delta BG}{8} \quad (37)$$

Here

ΔBG = change in bit tooth wear

It can be calculated based on the WOB, ROP, relative rock abrasiveness, and confined rock strength.

$$\Delta BG = W_c \sum_{i=1}^{n} WOB_i \, N_i \left(A_{r_{abr}} \right)_i S_i \quad (38)$$

Here

W_c = bit wear coefficient

$A_{r_{abr}}$ = relative abrasiveness

Rock compressive strength is a function of pressure and lithology:

$$S = S_o \left(1 + a_s P_e^{b_s} \right) \quad (39)$$

Here

S = confined rock strength

S_o = unconfined rock strength

a_s, b_s = coefficient depends on formation permeability

3.1.1.8 Pessier and Fear Method

Pessier and Fear (1992) elaborated the mechanical-specific energy methodology which was developed by Teale (1962). They performed simulator tests on the computer and conducted laboratory tests to quantify and develop an energy-balanced model for the drilling of boreholes under hydrostatically pressurized conditions. They derived an equation for mechanical specific energy, Eq. (40). They found better identification methodologies (than WOB and ROP concentrated evaluation) for bearing problems of the drill bits, which are quicker and more reliable by continuously monitoring Es and μ, Eq. (41).

$$E_s = WOB\left(\frac{1}{A_B} + \frac{13.33\,\mu_s\,N}{d_b\,ROP}\right) \quad (40)$$

$$\mu = 36\frac{T}{d_b\,WOB} \quad (41)$$

Here

Es = bit specific energy, psi

AB = borehole area, in^2

μ_s = bit specific coefficient of sliding friction

μ = apparent viscosity at 10,000 sec–1, cp

3.1.1.9 Osgouei Model

Osgouei (2007) developed a drilling model for predicting the ROP by considering the effect of the various drilling parameters. In his study, Bourgoyne & Young's model is improved and enhanced for both PDC and insert–tooth–roller bits as well as for horizontal and directional wells. The major improvements are the consideration of additional drilling parameters occurring due to inclination as well as the re-definition of the same drilling parameters due to PDCs. He initiated the model as:

$$ROP = (f_1)(f_2)(f_3)(f_4)(f_5)(f_6)(f_7)(f_8)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(f_n) \quad (42)$$

Where $f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8, \ldots\ldots\ldots f_n$ represent the functional relations between penetration rate and various drilling variables. Each of these functions contains constants which are shown as

a_1 through a_n. Determination of these constants is accomplished by using a multiple regression analysis of collected drilling data.

3.2 Drilling muds modeling

During a well drilling job, the mud is injected (at the surface) into the drill string (pipe) and returns to the surface through the annulus (outside the drill string) carrying cuttings generated by the bit. Although the pipe and annulus cross-sectional areas change along the well length, the geometry is usually simplified by assuming them as constant. The pipe and annulus are completely rigid and do not undergo any kind of strain.

A large majority of studies concerning the mathematical modeling of the drilling mud flows have involved various time-independent rheological models such as Bingham, Casson, Herschel–Bulkley, etc. More complex rheological models, such as the three-parameter Herschel–Bulkley model, are accepted to be more accurate than two-parameter models, such as the Bingham and the Casson models, in predicting the rheological behavior of drilling muds. However, complex mathematical models for drilling mud flow studies are not generally accepted because of the difficulties in finding analytical solutions for the fluid mass, momentum, and energy conservation equations and the complexity of the derivation of such parameters as the Reynolds number and the friction factor. Although in recent years, mathematical modeling and numerical simulation studies of multiphase flows in injecting or producing wells have made significant progress (Hasan and Kabir, 2002; Livescu et al., 2010), in general, this is not the case for drilling mud flows. Many studies concerning mathematical modeling of drilling mud flows involve empirical correlations and/or severe simplifications to identify different flow regimes (laminar, turbulent, and transitional between laminar to turbulent), calculate fluid velocity profiles, tubing, and annular pressure drops, etc. (Kelessidis et al., 2006; Reed and Pilehvari, 1993). A compromise between the accuracy of the model and the simplicity of the calculations has been the main driver for the mathematical modeling of drilling mud flows. It is still widely accepted that the best way to achieve this compromise, with the simplification that thixotropic or viscoelastic effects can be neglected, is to use the Herschel– Bulkley model.

Several authors contributed to the understanding of the rheological behavior of drilling muds flowing through wells. For example, Kelessidis et al. (2006) observed that the pressure drops and velocity profiles for laminar flow in pipes and annuli can be significantly affected by the choice of the rheological parameters.

Alderman et al. (1988) observed that when fitting the Herschel– Bulkley model with experimental data for water-based muds, high shear mud viscosity decreases with temperature similarly as does the aqueous phase viscosity and increases with pressure to an extent which increases with mud density. Also, the yield stress is independent of pressure and depends only weakly on temperature until a characteristic temperature is reached, then the yield stress increases approximately exponentially with the inverse of temperature. Last but not least, Alderman et al. (1988) observed that the flow behavior index n in Eq. (43) increases with temperature and decreases with pressure to an extent that increases with mud density.

$$\tau = \tau_y + k\dot{\gamma}^n \qquad (43)$$

where ty is the yield stress, τ_y, η_B and η_C are the Bingham and Casson viscosity, and k and n represent the consistency factor and the flow behavior index, respectively

Bailey and Peden (2000) presented a comprehensive suite of general formulations suitable for hydraulic calculations for conventional and unconventional (slimhole, underbalanced, and horizontal) wells. They assumed a time-independent three-parameter correlation for describing the drilling mud rheology and developed a generalized rheological parameter that couples laminar and turbulent flow regimes. Like Reed and Pilehvari (1993) before them, Bailey and Peden reduced the general set of flow and rheological equations for the drilling mud well flows to a set of correlations developed for steady-state, isothermal, incompressible, viscoplastic, single-phase fluids flowing through pipes and annuli (the annulus was modeled as a tube with an equivalent diameter).

Maglione et al. (2000) coupled the Herschel–Bulkley model for the mud rheology and the one-dimensional mass and momentum conservation equations inside a drilling well to compute the velocity profile, Reynolds number, and pressure drop, in both circular and annular sections. Maglione et al. made several simplifying assumptions for both rheologic and flow behaviors. The main assumptions for the Herschel–Bulkley model are that the drilling mud is incompressible and the rheological parameters are independent of pressure and temperature. The assumptions for the flow model include the following: steady state axial flow, concentric annulus and tubing sections, annular section considered as a rectangular slot, and laminar or turbulent flow only (plug flow has been considered as part of the laminar flow while transitional flow has been neglected).

Founargiotakis et al. (2008) also presented an integrated approach for the laminar, transitional, and turbulent flow of drilling muds in the concentric annulus, modeled as a slot. Their integrated model consisted of a set of analytical, semi-analytical, and empirical equations. Thus, they coupled the Herschel–Bulkley model for the drilling mud rheology with prior analytical solutions for laminar flow; correlations for turbulent flow, using the Metzner–Reed Reynolds number; and correlations for transitional flow, introducing transitional Reynolds numbers which depend on the local flow behavior index n.

Sorgun and Ozbayoglu (2011) developed a computational fluid dynamics model for predicting the frictional pressure drop during horizontal drilling in both concentric and eccentric annuli. They coupled the power-law model for the drilling mud rheology with one-dimensional, incompressible, single-phase mass and momentum conservation equations. This CFD model was validated against experimental data.

Billingham and Ferguson (1993) had a different approach to studying the bentonite mud (an inelastic, thixotropic, generalized-Newtonian fluid) well flow. The well was modeled as a pipe (no annulus) with a circular cross-section and the flow was one-dimensional and axisymmetric. They used Eqs. (44) and (45) to describe the bentonite mud rheology.

$$\frac{\partial \lambda}{\partial t} = a(1-\lambda) - b\lambda\dot{\gamma} + \mathcal{D} \qquad (44)$$

$$\tau = \mathrm{sgn}(\dot{\gamma})\{\lambda\tau_y + [\eta_\infty + (\eta_0 - \eta_\infty)\lambda]|\dot{\gamma}|^n\}, \quad |\tau| \geq \tau_y\lambda \qquad (45)$$

$$\dot{\gamma} = 0, \quad |\tau| < \tau_y\lambda$$

where D represents a structural diffusion term. Billingham and Ferguson showed that diffusion must be included in order to ensure that the model is structurally stable when applied to axisymmetric pipe flow.

The rheological model was not structurally stable, for certain parameter ranges, but this problem was eliminated by introducing the diffusion of the fluid-structure in Eq. (46)

$$\mathcal{D} = \frac{1}{r}\frac{\partial}{\partial r}\left[D^{*}(\lambda|\dot{\gamma}|)r\frac{\partial \lambda}{\partial r}\right] \qquad (46)$$

where r is the radial distance from the axis of the pipe and D is the rate of diffusion of fluid-structure. This rheological model was coupled with an isothermal, incompressible, single-phase momentum conservation equation. Billingham and Ferguson used two different sets of boundary conditions for the structural parameter λ at the pipe wall and showed that by using each of them obtained an identical solution at leading order when the rate of diffusion of the fluid-structure D is sufficiently slow.

Studying the solutions for λ, Billingham, and Ferguson showed that, for monotonic equilibrium rheograms, there is a unique, stable steady state solution and examined the approach to equilibrium. More interestingly, they also found that their model can give rise to non-monotonic rheograms. In this case, when no diffusion of fluid structure is allowed, multiple, stable, steady-state solutions exist. The equilibrium solution of the initial value problem depends on the details of the initial fluid structure. On allowing a vanishingly small rate of diffusion of fluid structure, Billingham and Ferguson also found that only one steady-state solution exists. This shows that, when the rheogram is not monotonic, the model is not structurally stable unless some diffusion of fluid structure is included.

Negrao et al. (2010) ~ presented a one-dimensional, isothermal, compressible (pressure-dependent density), single-phase mathematical model for the start-up flow of gelled drilling muds in annular spaces and circular pipes. They considered transient and advection terms in the momentum conservation equation. They used the Fannin friction factor used before by Chang et al. (1999) for the frictional pressure drop in laminar pipe flow and a different correlation (Fontenot and Clark, 1974) for the annular flow.

Although the studies listed above contribute to the understanding of drilling mud rheology and flow in wells, all of them assume severe simplifications which make their conclusions applicable to only a limited range of real downhole operating conditions. For example, the muds are generally considered incompressible and single-phase; the pressure and temperature effects on rheology or flow are usually ignored; transient effects are ignored on flow and may also be ignored on rheology; full multiphase mass, momentum, and energy conservation equations are not solved; finally, validation and verification of these theoretical flow models are usually ignored. All these severe simplifications prove that the current mathematical models for drilling mud well flow problems need to be re-evaluated.

The focus of this review paper is on the mathematical modeling of thixotropic drilling mud are presented in tabular form in Tables 3

Authors	Details
Houwen and Geehan (1986)	Rheological behavior and modeling of oil-based muds
Alderman et al. (1988)	High-temperature, high-pressure rheological behavior and modeling of water-based muds
Billingham and Ferguson (1993)	Modeling and simulation of thixotropic bentonite muds with yield stress
Pereira and Pinho (2002)	Rheological behavior and modeling of turbulent flow of thixotropic drilling muds with yield stress
Herzhaft et al. (2006)	Transient rheological and structural modeling of thixotropic oil-based muds with yield stress
Negrão et al. (2010)	Modeling and simulation of thixotropic drilling muds with yield stress in deepwater environments

Table 3 Studies involving mathematical modeling of thixotropic drilling mud flows

4. Conclusion

To effectively develop an optimal control system for the rotary drilling of oil wells, all the existing mathematical models for calculating the penetration rate would be analyzed critically. The comparative analysis of the drilling rate of penetration models is done with a focus on their application in the petroleum industry, analyzing the parameters of their methods as well as their benefits and weaknesses. The development of all these suitable models took place in the first 20 years of research into drilling optimization, such as rotary drilling bits, fluid dynamics, casing installation; and cement (Tansev, 2013). The earliest era is the period during which all methods and tools improved, hence it was named the development period (Tansev, 2013). This is the period that precedes the development of Speer's model.

This was followed by the scientific era in which oil companies started to perceive the importance of research. Speer, Graham and Muench, Maurer and Galle and Wood models were developed in this period (Tansev, 2013). During these years the scientific period took place and consequently the total cost increased (Tansev, 2013). The thought for optimized drilling is one of the most important assumptions of the scientific period. Speer, Graham and Muench, Maurer and Galle and Wood spent a lengthy time studying all parameters included in drilling and the relation between them.

This period was closely followed by an era known as automation period. Research by Reeds initiated this period. At that time the first computer systems were created which performed operations improving drilling (Tansev, 2013). Most of oil and gas companies started to use automated rig systems, based on closed-loop computer system that controlled drilling variables and had complete planning of well drilling from spud to production. Models by Bourgoyne and Young, Reza and Alcocer, Warren, Parker, Osgouei, John, Iqbal, Thomas were developed in this period.

The Graham and Muenh study in 1959 can be regarded as the first integrated model which approached and included the most important drilling factors (Maidlaand, 2010). More precisely this mathematical model evaluated the correlation between WOB and rotary speed, as well as the shelf life of bit (Maidlaand, 2010). In short, drilling rate was predicted combining depth, rotary speed and WOB (Maidlaand, 2010). Another research was carried out in 1963 by Galle and Woods in which they created special arranged graphs which indicated the best combination of drilling parameters. So far, the most important model on which all modern studies have relied is the linear penetration model by Bourgoyne and Young (Maidlaand, 2010). This model uses multiple regression analysis in order to achieve the best selection of drilling parameters.

The Speer's, Graham & Muench, Maurer, Galle & Wood, and Reed models are designed to achieve one-dimensional drilling and cannot be utilized for dynamic drilling. Bourgoyne and Young, Reza and Alcocer, Warren, Parker, Osgouei, John, Iqbal, and Thomas developed an inclusive system that can achieve multidimensional drilling which makes them suitable for dynamic drilling.

Reed's method predicted the best combination of factors, such as weight on the bit and rotary speed, taking into consideration two different cases, when all other variables were constant and when they have fluctuated. This method reached the same result as the Galle and Wood method but is considered more accurate because it has resolved the Monte Carlo Scheme. It should also be mentioned that this method presented effective advantages in connection with field application.

Viewing all the rate of penetration methods and comparing their strengths can help in deciding which model works best for a particular drilling operation to reduce cost. The works researched spanned from 1958 to 2018 cutting across the scientific research period and the automation period. the major comparison areas included method, year of discovery, period in history, effect on cost reduction, technique utilized, dimensional capacity, objective, drilling parameters considered, parameters not considered, mathematical equation, contribution to knowledge and what can be improved on.

Early models focused on modeling several parameters that influence the boiling rate while the other variables were assumed or retained. A complete and comprehensive simulation was subsequently given with most of the parameters that influence penetration levels. Optimization models have already been developed that can optimize the parameters influencing the penetration rate in real time

5. References

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