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Measuring the thermal conductivity of gases using boundary layer

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بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

﴿وَقُلِ ٱعۡمَلُواْ فَسَيَرَى ٱللَّهُ عَمَلَكُمۡ وَرَسُولُهُۥ وَٱلۡمُؤۡمِنُونَۖ ١٠٥﴾

سورة التوبة – الأية ١٠٥

الإهداء

بسم الله الرحمن الرحيم

"وَقُلِ ٱعْمَلُواْ فَسَيَرَى ٱللَّهُ عَمَلَكُمْ وَرَسُولُهُۥ وَٱلْمُؤْمِنُونَ"

نحمد الله تعالى حمدًا كثيرًا طيبًا مباركًا فيه، على ما وفقنا إليه من علم وعمل، وعلى نعمه التي لا تُعد ولا تُحصى، وكان هذا البحث ثمرة منها.

نهدي هذا الجهد المتواضع إلى:

- **أهلنا الأعزاء، الذين كانوا لنا العون والدعامة، في كل خطوة، وكل لحظة تعبٍ أو إنجاز... فلكم خالص الحب والتقدير والدعاء.**
- **مشرفنا الفاضل الدكتور أحمد كاظم الشرع، لما بذله من جهد في توجيهنا، ومتابعتنا، وإثراء أفكارنا بالعلم والصبر والحرص... فجزاه الله عنا خير الجزاء.**
- **الملاحظ مصطفى جبار ، لدوره المميز وجهده الواضح في الدعم الفني، الذي كان له الأثر الكبير في نجاح الجانب العملي من هذا البحث.**
- **المهندس رحيم النوري، لمساهمته الفاعلة ومساعدته المستمرة في التغلب على التحديات الفنية، فلن ننسى دعمه وإخلاصه.**

كما نهدي هذا العمل إلى كل من آمن بنا، ووقف إلى جانبنا بكلمة، أو نصيحة، أو دعاء.

والحمد لله رب العالمين، والصلاة والسلام على محمد وآله الطيبين الطاهرين.

Contents

Abstract

This research focuses on experimentally and theoretically determining the thermal conductivity of gases while accounting for the influence of the thermal boundary layer—a factor often overlooked in conventional measurements. A specialized experimental setup was designed, consisting of a bottom-heated rectangular duct with controlled airflow and heat flux. Surface temperatures were measured using contact sensors, and data were analyzed using Fourier's law to extract temperature-dependent thermal conductivity values. The mathematical modeling includes a third-order polynomial temperature profile within the thermal boundary layer, supported by boundary condition analysis. The study also evaluates various sources of heat loss, such as radiation, conduction, and convection, to refine the accuracy of the results. Experimental findings demonstrated noticeable deviations from theoretical values due to boundary layer effects. The research concludes with a discussion on the sources of error and limitations encountered during the experiment, providing recommendations for improving future thermal conductivity measurements in gases under non-standard conditions.

Chapter 1

1.1 Introduction

The thermal conductivity of gases is one of the fundamental physical properties that directly influences heat transfer across a wide range of industrial and scientific applications, from the design of insulating systems to advanced technological uses such as sensors and environmental scanning electron microscopes (ESEM). Researchers have shown increasing interest in studying the behavior of thermal conduction in gases, especially under non-traditional operating conditions such as low pressures, porous media, or confined gas-solid interfaces—where surface effects and phenomena like the thermal boundary layer become highly significant. Previous studies have demonstrated that heat transfer in gases is not solely governed by molecular collisions as assumed in classical Fourier-based models, but is also influenced by microscopic factors such as gas-surface interactions and the development of boundary layers at solid interfaces. The thermal boundary layer represents a distinctive thermal transition zone characterized by sharp thermal gradients and altered gas dynamic properties compared to the bulk flow. Ignoring this boundary layer can lead to inaccurate thermal conductivity measurements, particularly in experiments conducted in closed or microscale systems. Modern models—both experimental and theoretical—suggest that the boundary layer contributes to a reduction or deviation in the measured thermal conductivity by restricting the free movement of molecules near surfaces. Several researchers have attempted to characterize this effect either by modifying classical thermal equations or by modeling the interaction of physical mechanisms within narrow pores and interstitial spaces. Some studies have adopted a dual approach, combining precise laboratory experiments with numerical simulations based on energy and flow equations, thereby providing a deeper understanding of how environmental conditions affect heat transport in gases. In this context, the need emerges for an experimental study that explicitly accounts for the thermal boundary layer when evaluating the thermal conductivity of gases—a factor often overlooked in conventional measurements. The proposed experiment in this work aims to measure the thermal conductivity of selected gases using a specially designed apparatus to generate and monitor the boundary layer under controlled pressure, temperature, and medium composition conditions. Experimental results will be compared with theoretical values and reference models to quantify any deviations and interpret them in light of the boundary layer's properties. This study contributes to the advancement of thermal measurement techniques and a deeper understanding of complex phenomena related to heat transfer in gaseous media. It also paves the way for re-evaluating accepted thermal conductivity values in the literature, particularly in microscale applications or under non-standard conditions. The findings of this work help fill an existing knowledge gap in the field and may serve as a foundation for developing more accurate thermal models suited for modern engineering and scientific applications.

1.2 Objectives

1. To estimate the temperature-dependent thermal conductivity mathematically and experimentally.
2. To achieve accurate thermal property measurements using only surface temperature data, by leveraging numerical modeling of one-dimensional heat conduction, thereby minimizing the need for embedded thermal sensors within the sample.
3. To analyze the influence of the thermal boundary layer on heat transfer behavior in gases.

Chapter 2

2.1 Overview

This study focuses on the measurement and estimation of the temperature-dependent thermal conductivity particularly under the influence of thermal boundary layers. Accurate characterization of thermophysical properties in gases is essential in a variety of applications, including environmental scanning electron microscopy (ESEM), insulation systems, and confined gas environments. Traditional approaches to measuring gas thermal conductivity often fail to capture the subtle yet significant effects of boundary-layer phenomena, especially under low-pressure or microscale conditions.

The analysis accounts for key physical mechanisms occurring at the gas-solid interface, emphasizing the role of the thermal boundary layer as a resistive region that alters local heat transfer rates. By comparing filtered experimental data with theoretical predictions, the study quantifies the deviation attributable to boundary-layer effects and evaluates how these deviations can be minimized through improved experimental design. Sensitivity analysis and optimal experimental design techniques, such as D-optimality, are also used to guide the selection of sensor locations and heating durations.

2.2 Literature Studies

- **Estimation of Linearly Temperature-Dependent Thermal Conductivity:**

  Farzad Mohebbi and his team developed an efficient inverse analysis method to estimate thermal conductivity in steady-state heat conduction problems. Their approach uses sensitivity analysis and the conjugate gradient method to accurately recover non-constant thermal conductivity.

- **Measurement of Thermal Conductivity Using the Transient Hot-Wire Method:**

  Alessandro Franco developed an economical and efficient apparatus for measuring the thermal conductivity of non-metallic building materials (0.2–4 W/m·K). The system uses a Nickel alloy wire as a heat source and thermocouples for temperature measurement, achieving accuracy within 5% under optimal conditions

- **Simultaneous Estimation of Thermal Properties Using MEGA:**

  A. Imani and team used a modified genetic algorithm (MEGA) to estimate temperature-dependent thermal conductivity and heat capacity. Combining MEGA with the Levenberg–Marquardt method improved accuracy, achieving minimal error even with single-sensor measurements.

Chapter 3

3.1 Mathematical Model

Fourier's law of heat conduction

Steady Heat Conduction in Plane Walls Conduction is the transfer of energy from the more energetic particles of a substance to the adjacent less energetic ones as result of interactions between the particles. Consider steady conduction through a large plane wall of thickness $\Delta x = L$ and surface area A. The temperature difference across the wall is $\Delta T = T_2 - T_1$. Note that heat transfer is the only energy interaction; the energy balance for the wall can be expressed:

$$\dot{Q}_{in} - \dot{Q}_{out} = \frac{dE_{wall}}{dt} \qquad (1)$$

For steady-state operation,

$$\dot{Q}_{in} = \dot{Q}_{out} = const. \qquad (2)$$

It has been experimentally observed that the rate of heat conduction through a layer is proportional to the temperature difference across the layer and the heat transfer area, but it is inversely proportional to the thickness of the layer.

$$\text{rate of heat transfer} \propto \frac{\text{(surface area)(temperature difference)}}{\text{thickness}}$$

$$\dot{Q}_{Cond} = kA\frac{\Delta T}{\Delta x} \quad (W) \qquad (3)$$

The constant proportionality k is the thermal conductivity of the material. In the limiting case where $\Delta x \to 0$, the equation above reduces to the differential form:

$$\dot{Q}_{Cond} = -kA\frac{dT}{dx} \quad (W) \qquad (4)$$

which is called Fourier's law of heat conduction. The term dT/dx is called the temperature gradient, which is the slope of the temperature curve (the rate of change of temperature T with length x).

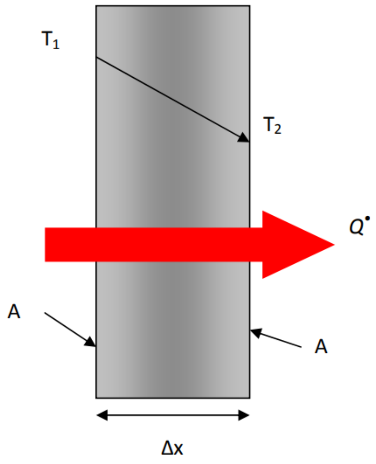

Figure 3.1: Heat conduction through a large plane wall.

Thermal Conductivity

Thermal conductivity k [W/m.K] is a measure of a material's ability to conduct heat. The thermal conductivity is defined as the rate of heat transfer through a unit thickness of material per unit area per unit temperature difference. Thermal conductivity changes with temperature and is determined through experiments.. An isotropic material is a material that has uniform properties in all directions.

Gas thermal conductivity

The present project aims to find the thermal conductivity of gases using the gas flow over a heated plate at constant heat flux as shown in the following figure.

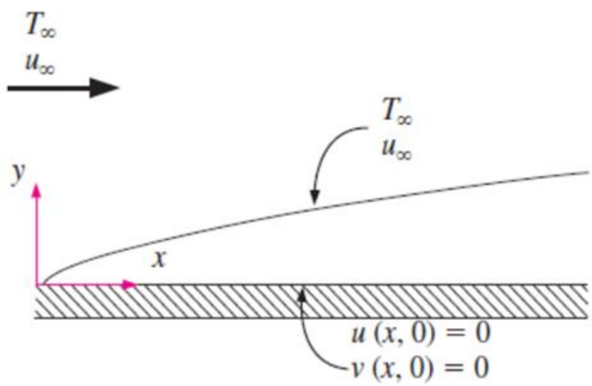

Figure 3.2: Boundary layer of external flow

The temperature distribution inside the thermal boundary layer is assumed as:

$$T(y)=A+ B\, y+ C\, y2+D\, y3$$

The boundary conditions are:

At $x=0$ $u(0,y)=U_\infty$ and $T(0,y)=T_\infty$

At $y=0$ $u(x,0)=0$, $v(x,0)=0$ and $q''=-k\, dT/dy$

At $y=H$ $u(x,H)=U_\infty$ and $T(x,H)=T_\infty$

Where U_∞ and T_∞ velocity and temperature at the free stream respectively. q'' and k constant heat flux (W/m^2) and thermal conductivity (W/m.K) respectively. H the height of flow (m).

Method of calculation gas thermal conductivity

After drawing the thermal boundary layer, measuring the temperature difference ΔT between the temperature of the wall and the nearest temperature ($\Delta y=0.5mm$) of the fluid then the thermal conductivity of gas equal to,

$$k = \frac{q'' \,\Delta y}{\Delta T}$$

Chapter 4

Experiential Setup

4.1 General Description

The objective of the experimental study is to find the thermal conductivity of gases. In this chapter, a description of the experimental setup is presented.

4.2 Experimental Apparatus

4.2.1 Gas Duct

Air was used as the gas and driven by the induced draft fan. The duct with dimensions 20×30×10 cm^3. The flow is laminar flow conditions with inlet velocity at the duct entrance.

4.2.2 The test section

The test section is a bottom heated rectangular duct. It consists of a 20×30×0.6 cm^3 aluminum plate. The section view of the rectangular duct is schematically shown in Fig. 4.1 section (A-A). As can be seen from this figure, the test section of the duct was insulated with 5 cm glass wool and wooden box on all sides in order to direct all heat to the surface.

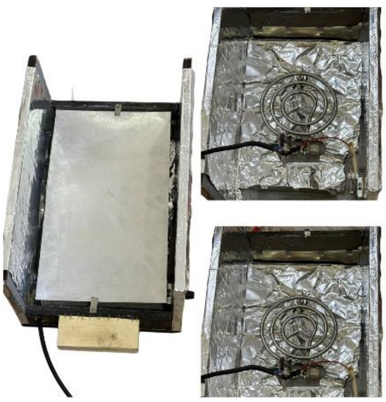

Figure 4.1 : Experimental Apparatus

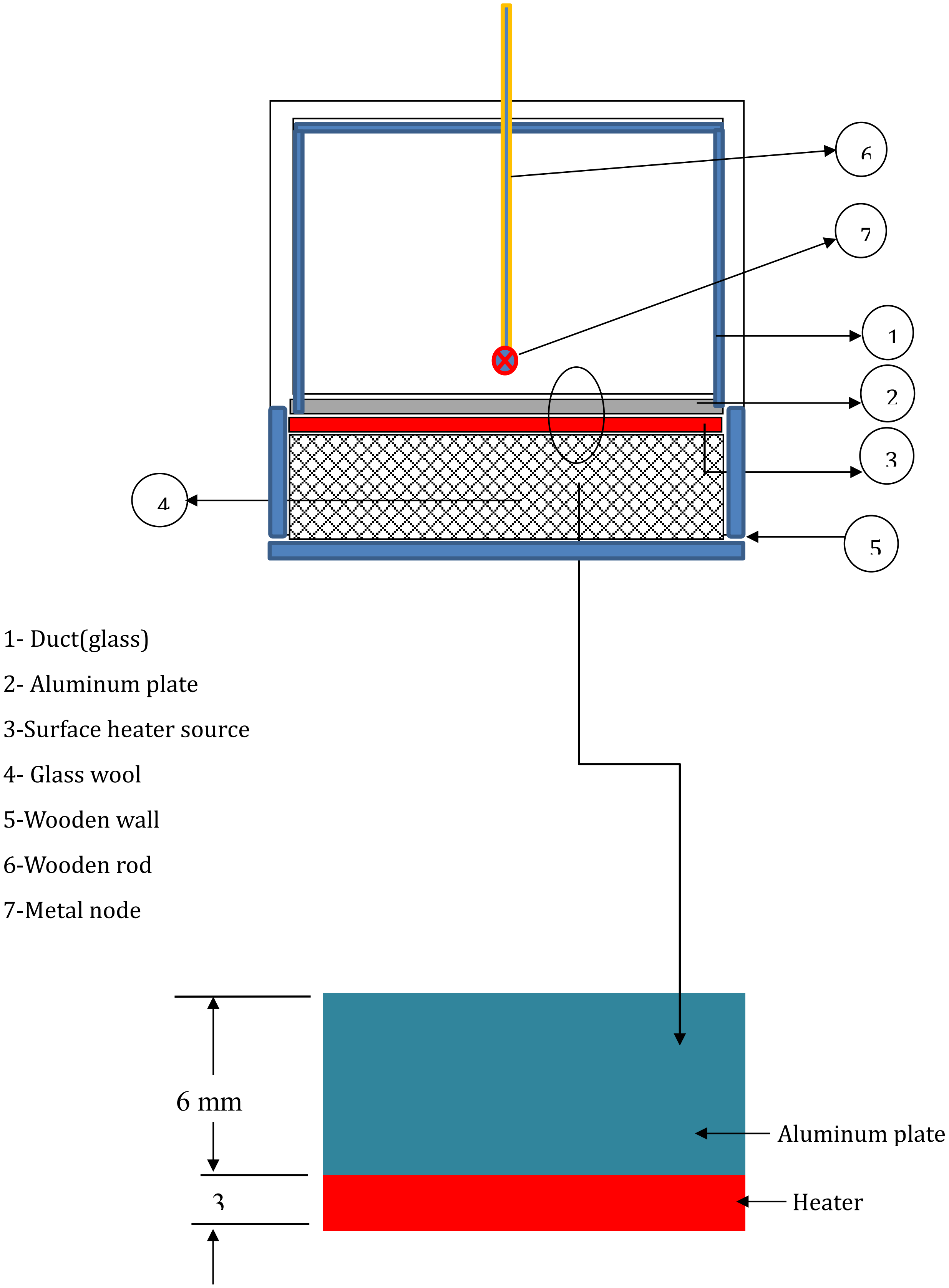

Figure 4.2: *Section A-A.*

4.2.3 Heater circuit

The bottom wall of the test section was made of 6 mm thick aluminum plate that heated using a electric heater. A sheet heater plate was placed under the aluminum plate, with a size equal to the size of aluminum plate having dimensions of 20 cm×30 cm.

Electric current was provided to the heater plate via a electric source providing a heat flux boundary condition specified for a decided experimental case. The heater is placed in contact underneath of the base, The diagram of the heater circuit is shown in Fig.4.2 .It consist of heater, the digital voltmeter which connected in parallel to the heater, and the digital ammeter which connected in series to the heater. The input voltage and current densities were measured separate voltmeter and ammeter.

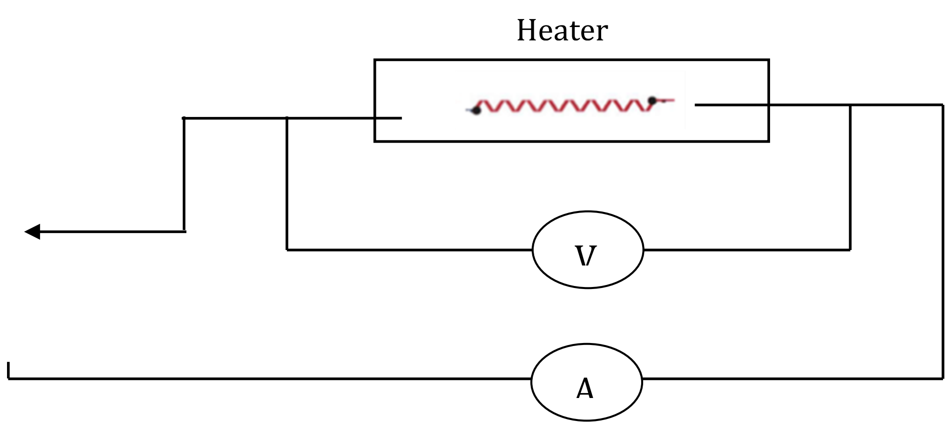

Figure 4.3 : *Electrical circuit of the heater.*

4.2.4 Ammeter

A ammeter is a device used to measure the intensity of the electric current in a circuit, and always connects respectively with the elements through which the current passes. Its work depends on the principle of the magnetic effect of the electric current, and its internal resistance is very small so as not to affect the value of the current to be measured. The ammeter is used in many electrical and electronic applications, whether in laboratories or in industrial devices, to determine the efficiency of the circuit or detect faults

4.2.5 Voltmeter

A voltmeter is a device used to measure the voltage difference between two points in an electrical circuit, and always connects in parallel with the element on which the voltage is to be measured. It has a high internal resistance to reduce its impact on the circuit during measurement. The voltmeter is used in various electrical and electronic applications, whether in education or industry, to monitor the performance of devices and ensure the safety of electrical connections

4.2.6 Mercury thermometer

A mercury thermometer is an instrument used to measure temperature, and its work depends on stretching mercury inside a fine glass tube when the temperature changes. This type of type is characterized by accuracy and the ability to measure temperatures within a specific range with high accuracy, and has been used for many years in the medical and scientific fields. Although it has become less common today due to concerns about mercury toxicity, it is still considered an exact classic in temperature

4.3 Experimental procedure

The following procedure was followed to carry out an experiment run:

1. Switch on the main switch of power supply of the apparatus.
2. Switch on the fan.
3. Switch on the electrical heater and the heater input power then adjusted to give the required heat flux.
4. The apparatus was left at least 15 min to establish steady state condition, then using the thermometer to read the temperatures in two specify points.
5. The steps from 1 to 4 can be repeated for another gas.

During each test run, the following readings were recorded:

a. The reading of temperature.
b. The heater current in amperes.
c. The heater voltage in volts.

4.4 Calculations

The total dissipated energy was determined from ohm's law,

$$Q = VI$$

The voltage drop V and current I were measured during the experiment. The heat is

$$k = \frac{Q/A}{\Delta T/\Delta Y}$$

Where **k** is the thermal conductivity of the gas. **ΔT** is the difference between surface and node temperature. **ΔY** is the distance between the surface and the node.

Chapter 5

Results and Discussions

5.1 Theoretical part

The temperature distribution is

$$\boldsymbol{T_{(y)} = A + By + Cy^2 + Dy^3}$$

$$\frac{dt}{dy} = B + 2Cy + 3Dy^2 \qquad (first\ drevetiv)$$

$$\frac{d^2T}{dy^2} = 2C + 6Dy \qquad (second\ drevitiv)$$

At Boundary condition :

$\mathbf{B.C.1}$: $\frac{d^2T}{dy^2} = 0 \quad \Rightarrow \quad \mathbf{C = 0}$

$\mathbf{B.C.2}$: $q = -k\frac{dt}{dy}_{y=0} = 0 \quad \Rightarrow \quad \boldsymbol{B = \frac{-q}{k}}$

$$T_{(y)} = A - \frac{q}{k} + Dy^3$$

$\boldsymbol{B.C.3}$: $\boldsymbol{at}$ $\frac{dt}{dy}_{y=\delta} = 0$

$$D = \frac{-q}{3K\delta_t{}^2}$$

$\boldsymbol{B.C.4}$: $T = T_\infty$ $\boldsymbol{at}$ $y = \delta_t$

$$A = T_\infty + \frac{2}{3}\frac{q}{K}\delta_t$$

Sub the values of (A,B,C,D) at the equation of The temperature distribution :

$$T_{(y)} = T_\infty + \frac{2}{3}\frac{q}{K}\delta_t - \frac{q}{k}y + \frac{q}{3K\delta_t{}^2}y^3$$

$$\boldsymbol{T_{(y)} = T_\infty + \frac{q}{k}\left(\frac{2}{3}\delta_t - y + \frac{y^3}{3\delta_t{}^2}\right)}$$

After extracting the values of **dy** and **dt** from the plot, and calculating the values of **k** using Fourier's law.

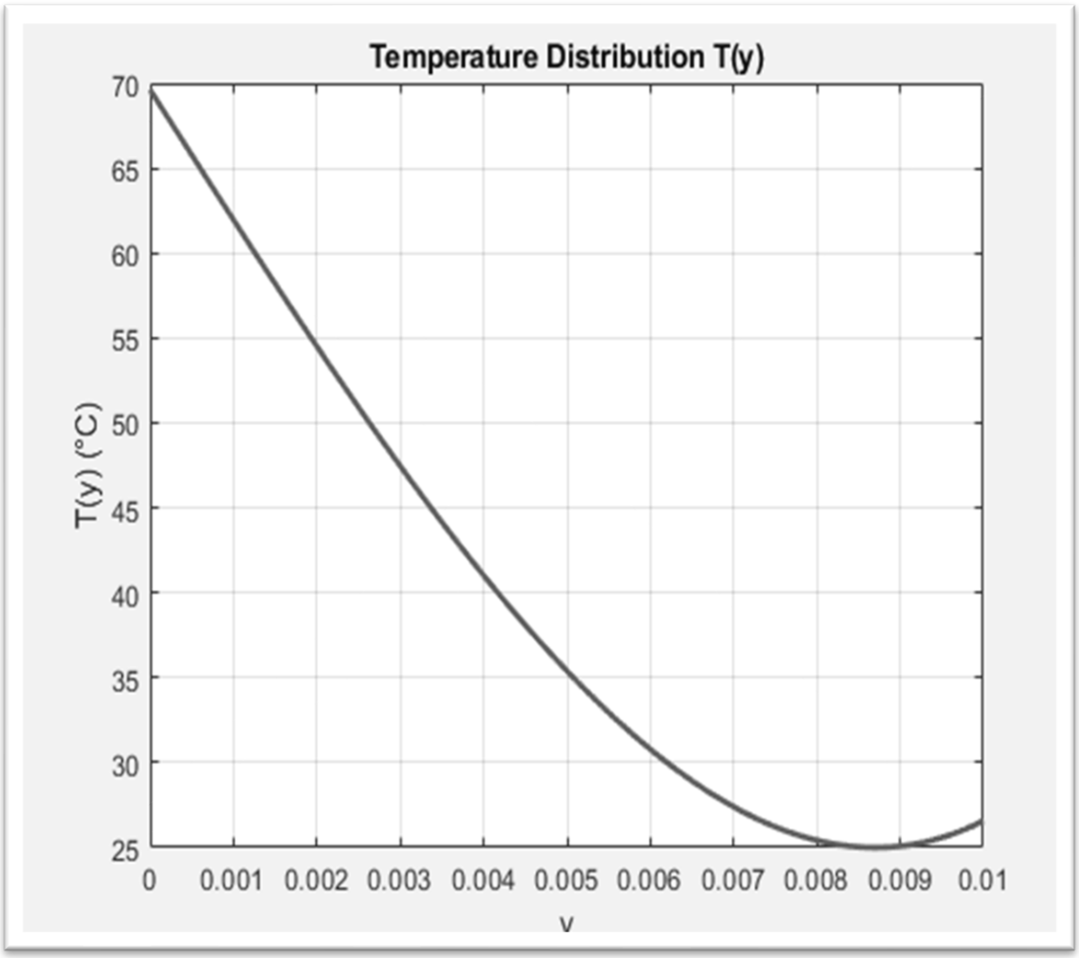

Figure 5.1 : Temperature distribution

Table (1): Theoretical Thermal Conductivity Compared with Reference Values

dy	K_{acl}	K_{cal}	$Error$
0.005	0.026	0.02365	0.0989
0.001	0.026	0.02683	0.03
0.0015	0.026	0.032	0.187
0.002	0.026	0.074677	0.6518

We obtain the least error at the second thickness, which is 0.001, so we will adopt it and disregard the others.

5.2 Experimental part

The heat losses in the experiment are:

1. **Radiation**

$$\boldsymbol{q_r = \varepsilon\sigma A(T_s^4 - T^4 sur)}$$

Case 1

T_s=107°C

T_{sur}=24°C

A=0.2*0.35=0.07m^2, σ=5.67*10^{-8} $w\ /\ m^2 \cdot k^4$

ε=0.4 or 0.2 for Aluminum oxidized

$q_r = 0.4 *$5.67*10^{-8}*0.07($380^4 - 297^4$)=20.75 w

2. **Conduction from wood with Ambient**

$$\boldsymbol{q = K \cdot A \frac{\Delta T}{t}}$$

A $= 0.07m^2$, $k = 15$,$t = 1cm$=0.01m

Assume $\Delta T = 107 - 24 = 83$°C

$$q = 0.05 * 0.07 * \frac{83}{0.01} = 87.15\ w$$

3. **Convection leakage from side walls**

Strip area = (2 * 0.35) * 0.01=0.007m^2

$q_{conv} = hA_{strip} * \Delta T$, $\Delta T = T_{sur}$-T_{amb}

Maximum (h) for gas reaches to

Assume h =250 $w / m^2 \cdot k$

$q_{conv} = 250 * 0.007 * (104 – 24) = 145.25\, w$

- Wires losses = 5 %

$q = \bar{I} * v = 220 * 3.85 = 847\, w$

- Wires losses = 0.05 * 847 = 42.35 w

$q_{total} = q_{rad} + q_{cond} + q_{conv} + q_{wires}$

=20.75 + 87.15 + 145.75 + 42.35 =295.5 w

% Of losses = $\frac{295.5}{847} \approx 35\ \%$

$\frac{847*0.65}{0.07} = k\frac{83}{0.0005}$

k=0.0473 $w / m^2 \cdot k$

5.3 Practical part

$q = kA\frac{\Delta t}{\Delta x}$

$k = \frac{q*\Delta x}{A*\Delta t}$

1)when $T_1 = 164$ ℃ , T_2 =60 ℃

ΔT=104

k=$\frac{(3.85*220*0.65)*0.0005}{0.2*0.35*104}$=0.037 $w / m^2 \cdot k$

2) $T_1 = 170$ ℃ , $T_2 = 63$ ℃

ΔT=107 ℃

k=$\frac{(3.85*220*0.65)*0.0005}{0.2*0.35*107}$=0.036 $w\ /\ m^2 \cdot k$

3) $T_1 = 180$ ℃ , $T_2 = 60$ ℃

ΔT=120 ℃

k=$\frac{(3.85*220*0.65)*0.0005}{0.2*0.35*120}$=0.032 $w\ /\ m^2 \cdot k$

4) $T_1 = 195$ ℃ , $T_2 = 71$ ℃

ΔT=124 ℃

k =$\frac{(3.85*220*0.65)*0.0005}{0.2*0.35*124}$=0.031 $w\ /\ m^2 \cdot k$

5) $T_1 = 205$ ℃ , $T_2 = 65$℃

ΔT=140 ℃

k =$\frac{(3.85*220*0.65)*0.0005}{0.2*0.35*140}$=0.028 $w\ /\ m^2 \cdot k$

$k_{ave}\ \frac{\sum k}{n}$=$\frac{k_1+k_2+k_3+k_4+k_5}{5}$=0.0328

$error = \left|\frac{0.0328-0.027}{0.027}\right| * 100 = 21.4\%$

Table (2): Experimental Thermal Conductivity Compared with Reference Values

dt	K_{acl}	K_{cal}	$Error$
104	0.027	0.037	0.37037
107	0.027	0.036	0.333
120	0.027	0.032	0.185
124	0.027	0.031	0.148
140	0.027	0.028	0.037

The experimentally measured thermal conductivity values of air ranged from 0.028 to 0.037 W/m·K, with an average of 0.0328 W/m·K, compared to the reference value of 0.027 W/m·K, resulting in an error rate of 21.4%. This deviation is attributed to heat losses through radiation, conduction, and convection, as well as the influence of the thermal boundary layer and limitations in measurement accuracy. Nevertheless, the results are considered acceptable within the experimental environment, indicating the effectiveness of the practical setup used.

5.4 Error

1. **Random error**

   is an unpredictable variation in measurement results due to uncontrollable external or human factors. It can be reduced by repeating measurements and averaging to improve accuracy

2. **Human error**

   is an error resulting from the actions or omissions of the person performing the measurement, such as an incorrect reading of a device or a timing delay. It can be reduced with training and focus during the experiment.

3. **Instrument error**

   is a malfunction resulting from defects or inaccuracies in the performance of a measuring instrument, such as an incorrectly calibrated or outdated instrument. It can be reduced by regularly maintaining and calibrating the instruments.

4. **A truncation error**

   occurs when numerical values are rounded or shortened during calculations, such as ignoring decimal places. It can be reduced by using more precise numerical methods and increasing the number of digits used in calculations.

5.5 Limitations and Experimental Challenges

- **Glass**

Due to exposure to a high heat source, the glass experienced thermal shock and subsequently broke. We then replaced the glass with wooden panels coated with a layer of heat-resistant silicone, followed by a layer of heat-resistant cellophane.

- **The fiberglass**

Due to the buildup of heat inside the wooden box, the fiberglass insulation was exposed to high temperatures, which caused it to burn and fail to withstand the heat. To solve this problem, we covered the fiberglass with a layer of heat-resistant cellophane.

- **The electrical wire**

Although the electrical wires used were heat-resistant, they were still burned. Therefore, it is recommended to use heat-resistant wire sleeving.

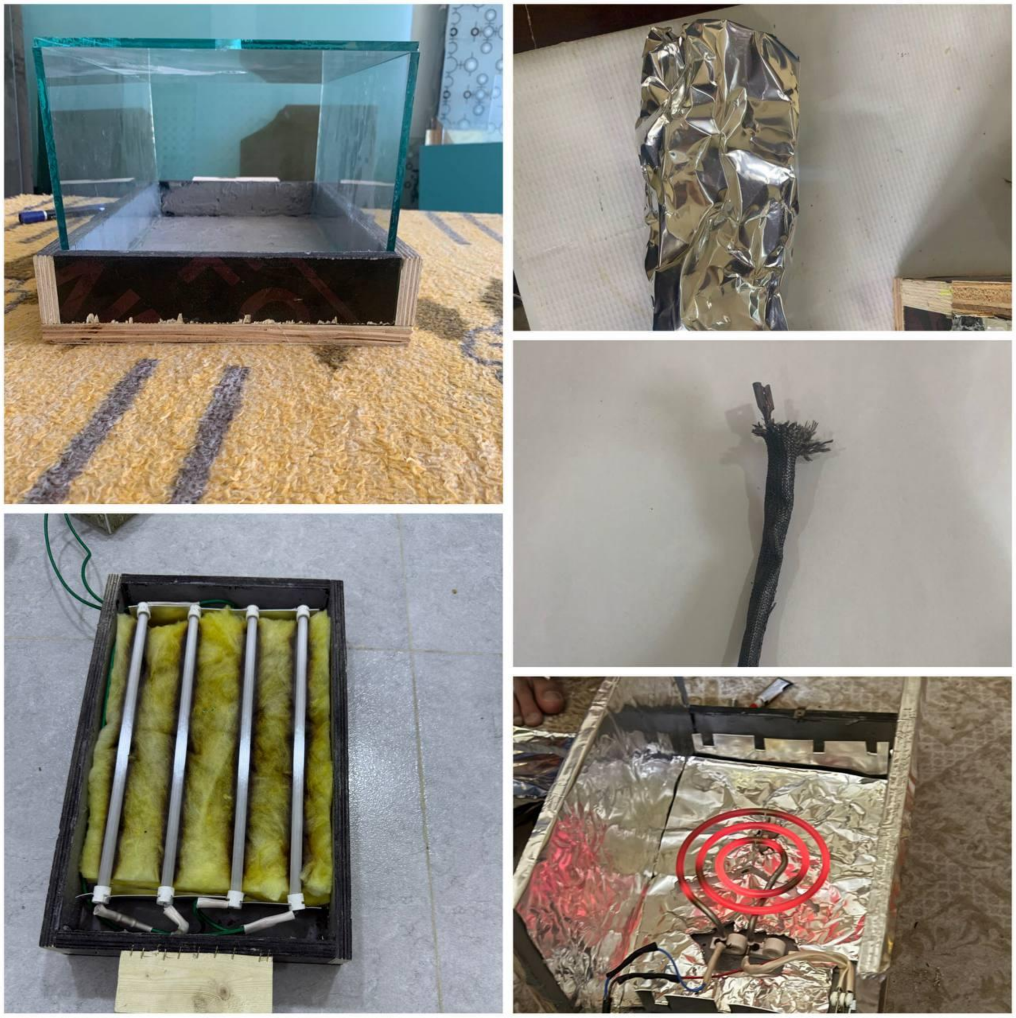

Figure 5.2: Limitations and Experimental Challenges

5.6 Project Conclusion:

At the end of this project, we successfully studied and understood the thermal conductivity properties of air by designing and implementing a practical system to measure it accurately. This project allowed us to apply theoretical concepts related to heat transfer and thermal conductivity, and connect them to a real-world experiment using appropriate measurement tools and techniques.

Through this experience, we gained insight into the challenges of measuring materials with low thermal conductivity, such as air, and how to improve measurement accuracy through insulation, temperature control, and minimizing heat losses. We hope this work contributes to future studies on thermal properties of materials and paves the way for industrial or academic applications in areas such as cooling, thermal insulation, and energy engineering.

In conclusion, we would like to express our sincere gratitude to everyone who supported and guided us throughout this project. We look forward to continuing to develop our skills in research and practical applications.

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