Building an InvertedTopp-Leone-Exponentinal Probability Distribution with Practical Application

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Abstract:

The process of fitting distributions is one of the common and well-known models for generating new distributions, which are complex distributions. In this research, a methodology was used to build a new proposed model of the Inverted - Topp - Leone distribution and the Exponential distribution , which is the transformative methodology (Transformed-Transformer Method) It is)symbolized by (TX family) to be a compound called((Inverted Topp Leone- exponentinal). With two parameters, the measurement parameter ϴ and the figure parameter λ , studying its statistical properties and estimating the reliability function using the Maximum Likelihood Method and applying it to real data represented by the operating times of electrical transformers until failure.

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The main terms of the research : Inverted - Topp -Leone ,methodology(TX Family) , (Inverted - Topp -Leone-Exponential)and Maximum Likelihood Method (MLE) .

to the(TransformedTransformer Method) or (theTX Family) which depends , methodology (on two random variables, one known as the transformed and the other as thetransformer to build a new probability distribution with two parameters. It is composed of two different distributions:Inverted - Topp -Leone as the base distribution and the supporting (distribution is theExponential distribution so member of the that this distribution is a new TX family .

) The distributionInverted -Topp -Leone Exponential is not constructed , its properties (are recognized and its parameters are estimated using estimation methods such as theMaximum Likelihood Method symbolized ,

Introduction:

There are many probability distributions in statistics that have been identified, studied and entered in many areas of life using real data, t we may face in some data problems for not bu reaching realistic results because the probability distribution is unable to dub the data and so we use other methods, namely Mixing or fitting distributions , where compound distributions are used to facilitate the process of analyzing data better than the distributions are single. The installation process can be applied to discontinuous and continuous probability distributions, and they can be installed together according to certain methods for the conditions .There are several synthesis process, but we will limit ourselves holidays, especially in electrical transformers, so we need distributions that are more harmonious to solve failure problems.

Research goal:

Distribution proposal Probabilistic (Inverted Topp Leone- exponentinal) through a methodology The Transformed -Transformer Method to know its suitability to the real data , Derivation of the general mathematical properties of a distribution The probability and dependency function for them by the methods, which is (the Maximum Likelihood Method).

mby the acrony"MLEM" and applied to real data.

Research problem:

The problem of the research is summarized in both theoretical and applied aspects. On the theoretical side, it was noted that there is a lack of writing according to the Transformed-Transformer Method . It must be shed light on it to build a new probability distribution, in addition to the problem of representing real data represented by working times until failure. On the practical side, most devices display To stop (failures) as a result of sudden

Theoretical side :

Topp_leone Inverted Distribution[1]

The top-leon distribution transformer is among the transformed distributions that are used in Many fields and applications including biological sciences, life test problems, survey sampling ..... etc. To find an Inverted Topp_leon Distribution Suppose Z is a random variable that follows the (TL) Topp_leon distribution Distribution :

$$f_{TL}(z) = 2\theta z^{\theta-1}(1\text{-}z)(2\text{-}z)^{\theta-1} \;;\quad 0 \le z \le 1 , \theta > 0$$

$$F_{TL}(z) = z^{\theta}(2\text{-}z)^{\theta} \;;\quad 0 \le z \le 1 , \theta > 0$$

(pdf) ndThe probability density function was fou$T = \frac{1}{z}$(the transformation) and by taking

T~ITP(θ):that is

$$f(t) = 2\theta t(1 + t)^{-1-2\theta}(1 + 2t)^{-1+\theta} t \ge 0 , \theta > 0 \quad (1)$$

: whereas

represents a random variable :t

represents the distribution parameter :θ

:is written as(CDF) The cumulative distribution function

$$F(t) = 1\text{-}\frac{(1+2t)^{\theta}}{(1+t)^{2\theta}} t \ge 0 , \theta(\quad 2)$$

Exponential Distribution[2]

The exponential distribution is one of the continuous statistical distributions of great importance in probability theory, and it has many statistical applications, especially in the fields of waiting queues, ic processes . ... etc., and the reason for this name is that the distribution reliability theory and stochast :depends on an exponential mathematical equation. And its mathematical formula is

x~E(λ)

$$f(x) = \lambda e^{-\lambda x} \quad ; x \in (0, \infty), \;\lambda > 0 \quad (3)$$

: whereas

random variable :X

distribution parameter :λ

:And the aggregate distributive function is

$$F(x) = 1\text{-}e^{-\lambda x} (4)$$

(Reliability function)Reliability function[7][8]

The .(t>0)t Define the reliability function as the probability that a machine will not fail for time broad meaning of the reliability function is a measure of the machine's performance. It is denoted by and indicates F(t) is a random variable with a probability distributionT we assume ,R(t) the symbol :tion can be expressed mathematicallythe time of failure, and the reliability func

R(t) = P (T > t) (5)

:And the aggregate function

R(t) = 1 – F (t) (6)

TransformedTransformerMethodOr Methodology (TX Family)[5][6]

Suppose that(X) represents the random variable and it is called thetransformer which transforms , another random variable, which is(T) which is called the transformed ,, and it generates a new (probability density functionpdf) and thus it is called the (TX Family methodology, where this methodology provides building distributions using Weight functionW(F(x)))weight function . suppose that ,To get the new cumulative distribution function(X) a random variable has a probability density functionf(x) and a cumulative distribution functionF(x) and that the random variable ,(T) has a probability density functionf(t) and for the period$[-\infty < a < t < b < \infty]$ the cumulative function , G(x) :for the new resulting family will be as follows

$$G(x) = \int_0^{W(F(x))} Z(t)dt \quad (7)$$

:The above equation can be written into the following form

G(x)= Z[W(F(x))] (8)

new resulting family can be found according to the g(x) The probability density function for the :formula following

$$g(x) = Z\{W(F(x))\} \times \{\frac{\partial}{\partial x} W(F(x))\} \quad (9)$$

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(T) of the random variable(pdf) function :Z(t)

which has the same properties as the ,(X) represents the function of the random variable :W(F(x))

:weight function that satisfies the following conditions

1- ((x) ∈ [b]

is monotonically differentiable and non-decreasing((x))2-3- ((x)) → $a as x \to -\infty$ and ((x)) → $b as x \to \infty$.

InvertedTopp_leone_Exponential Distribution (ITLED)

$$g(x) = h(x)z(H(x))$$

$$g(x) = h(x)z(-\log(1-F(x)))$$

$$g(x) = \lambda[2\Theta(-\log(1-(1-e^{-\lambda x}) \times [1 + (-\log(1-(1-e^{-\lambda x})))]^{-2\Theta-1} \times [1 + 2(-\log(1-(1-e^{-\lambda x})))]^{\Theta-1}$$

:Since

$$-\log(1-(1-e^{-\lambda x})) = -\log(1-1 + e^{-\lambda x})$$

$$-\log(e^{-\lambda x}) = \lambda x$$

:Substitute into the equation, so it becomes

$$g(x) = 2\theta\lambda^2 x(1 + \lambda x)^{-2\theta-1}(1 + 2\lambda x)^{\theta-1}; \infty < x < 0, \theta, \lambda < 0 \quad (10)$$

As for the aggregate function for the complex distribution (ITLE), it can be found:

$$G(x) = \int_0^x g(u)du \quad (11)$$

$$G(x) = 1-(1 + \lambda x)^{-2\theta}(1 + 2\lambda x)^{\theta}; \infty < x < 0, \theta, \lambda < 0 \quad (12)$$

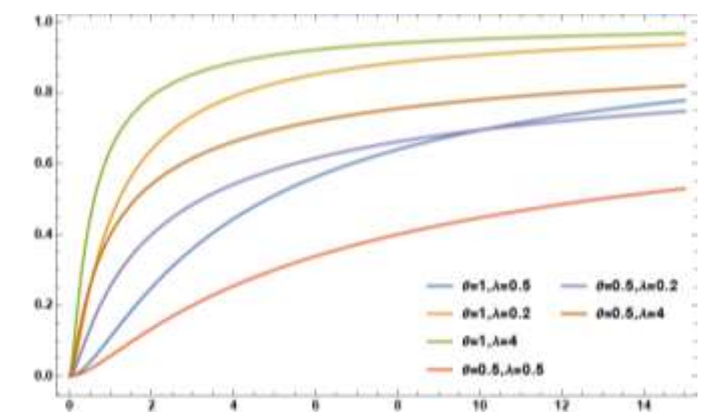
The reliability function of the complex distribution (ITLE) Reliability function [R(x)] To find the dependency function of the distribution, it is:

R(x) = 1 – G (x) (13)

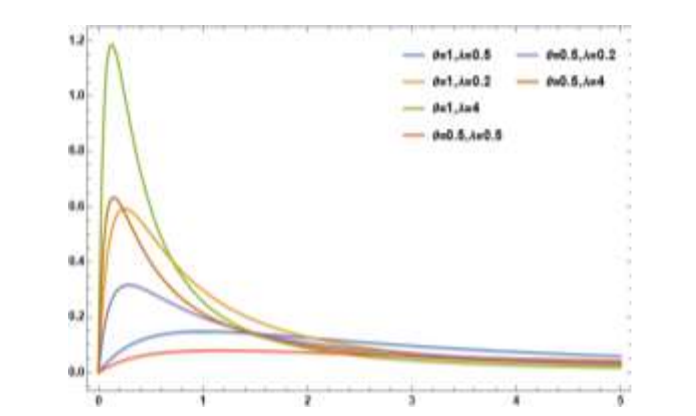
:the result is (13) .In formula No(12) .And by substituting formula No

$$R(x) = (1 + \lambda x)^{-2\Theta}(1 + 2\lambda x)^{\Theta} \quad (14)$$

(CDF of I.T.L.E)



(PDF of I.T.L.E)



$$\acute{\mu}_r = E(x^r) = \frac{2^{j+1}\theta\lambda^{2+j+k}}{r+j+k+2}\sum_{j=0}^{\theta-1}\sum_{k=0}^{-2\theta-1}\binom{\theta-1}{j}\binom{-2\theta-1}{k}\int_0^{\infty} x^{r+j+k+2}dx \quad (15th)$$

Methods Estimation

methodEstimation (MLE) The Maximum Likelihood

$$L(x_1, x_2, \ldots\ldots x_n, \lambda, \theta) = g(x_1, \lambda, \theta), g(x_2, \lambda, \theta) \ldots\ldots g(x_n, \lambda, \theta)$$

$$L(x_i, \lambda, \theta) = \prod_{i=1}^{n} g(x_i, \lambda, \theta)$$

:into the above formula, it will be (ITLE) the probability density function

$$L(x_i, \lambda, \theta) = \sum_{i=1}^{n}\left[2\theta\lambda^2 x_i(1+\lambda x_i)^{-2\theta-1}(1+2\lambda x_i)^{\theta-1}\right]$$

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$$L(x_i, \lambda, \theta) = (2\theta\lambda^2)^n \sum_{i=1}^{n} x_i(1+\lambda x_i)^{-2\theta-1}(1+2\lambda x_i)^{\theta-1}$$

:We take the logarithm of both sides of the above equation, so it becomes

$$\log L = n\log(2\theta\lambda^2) + \sum_{i=1}^{n}\log x_i + (-2\theta-1)\sum_{i=1}^{n}\log(1+\lambda x_i) + (\theta-1)\sum_{i=1}^{n}\log(1+2\lambda x_i)$$

$$= n\left(\log(2) + \log(\theta) + \log(\lambda^2)\right)\sum_{i=1}^{n}\log x_i - 2\theta\sum_{i=1}^{n}\log(1+\lambda x_i) - \sum_{i=1}^{n}\log(1+\lambda x_i)$$

$$+\,\theta\sum_{i=1}^{n}(1+2\lambda x_i) - \sum_{i=1}^{n}(1+2\lambda x_i) \qquad (16)$$

and set it λ) and θ) parameters for the Then it takes the first partial derivative of the above equation :zero, so it becomes to

$$\frac{\partial \log L}{\partial \theta} = \frac{n}{\theta} - 2\sum_{i=1}^{n}\log(1+\lambda x_i) + \sum_{i=1}^{n}\log(1+2\lambda x_i) = 0$$

$$\frac{\partial \log L}{\partial \lambda} = \frac{2n}{\lambda} + (-2\theta-1)\sum_{i=1}^{n}\log(1+\lambda x_i) + (\theta-1)\sum_{i=1}^{n}\log(1+2\lambda x_i)$$

$$\frac{2n}{\hat{\lambda}} - \frac{2\theta\sum_{i=1}^{n}x_i}{(1+\hat{\lambda}x_i)} + \frac{2\theta\sum_{i=1}^{n}x_i}{(1+2\hat{\lambda}x_i)} - \frac{\sum_{i=1}^{n}x_i}{(1+\hat{\lambda}x_i)} - \frac{2\sum_{i=1}^{n}x_i}{(1+2\hat{\lambda}x_i)} = 0$$

$$2\theta\hat{\lambda}^2\sum_{i=1}^{n}x_i^2 - 3\hat{\lambda}\sum_{i=1}^{n}x_i - 2n = 0$$

:respectively are (λand Θ So the estimated values of the parameters (

$$\hat{\Theta}_{ML} = \frac{n}{2\sum_{i=1}^{n}\log(1+\lambda x_i) - \sum_{i=1}^{n}\log(1+2\lambda x_i)}$$

$$\hat{\lambda}_{ML} = \frac{3+\sqrt{16\Theta+9}}{4\Theta\sum_{i=1}^{n} x_i}$$

The maximum possible estimator of the dependency function can be obtained by substituting the :estimators, so the formula is as follows

$$R(x) = (1+\hat{\lambda}_{ML}x)^{-2\hat{\Theta}_{ML}}(1+2\hat{\lambda}_{ML}x)^{\hat{\Theta}_{ML}}$$

[6](Goodness of fit tests)

fit

The data were tested using Mathematica program to show their suitability for the ITLED distribution . Goodness of fit tests was conducted using Anderson - Darling tests .Cramér - von Mises and **:**Kolmogorov-Smirnov) and their formulas are as follows

1. Anderson-Darling statistic :

$$A_d^* = n\sum_{i=0}^{n}\frac{[F_n(x)-F(x)f(x)]}{F(x)[1-F(x)]} \qquad (17)$$

2. Cramer -von Misesstatistic:

$$W_d^* = n\sum_{i=0}^{n}[F_n(x)-F(x)f(x)] \qquad (18)$$

3. Kolmogorov-Smirnov statistic:

$$D_d^* = \sum_{i=0}^{n}|F_n(x)-F(x)| \qquad (19)$$

:Since

$F_n(x)$:empirical distribution function represents the

.mentioned tests-According to the following hypothesis and for all the above

H_0: the data are distributed

H_1: the data are not distributed

Application side:

data datareal

the purpose of proving the suitability of the proposed distribution (ITLED), it will be applied to real data represented in the working times until failure of the private electrical transformers in the Rusafa area in Baghdad governorate. 9911

In order to find a solution to the problem (malfunction of electrical transformers), (95) transformers were registered during the period of work until the failure (damage) from the General Company for Baghdad Electricity Distribution , where the data were from January 2018 Until August 2021 for separate areas in Rusafa and for the purpose of knowing the carrying capacities of the transformers without any failure.

The following table shows the real data of the transformers until failure:

Table No. (1)

t_i	t_i	t_i	t_i	t_i	t_i	t_i	t_i	t_i	t_i
0.10	**0.10**	**0.14**	**0.23**	**0.29**	**0.32**	**0.40**	**0.40**	**0.48**	**0.52**
0.55	**0.61**	**0.64**	**0.69**	**0.70**	**0.73**	**0.74**	**0.74**	**0.75**	**0.76**
0.77	**0.79**	**0.80**	**0.80**	**0.87**	**0.89**	**0.94**	**0.98**	**1.00**	**1.00**
1.00	**1.10**	**1.10**	**1.20**	**1.20**	**1.20**	**1.20**	**1.30**	**1.40**	**1.45**
1.47	**1.57**	**1.60**	**1.70**	**1.78**	**1.78**	**1.80**	**1.80**	**1.86**	**1.90**
2.10	**2.10**	**1.16**	**1.28**	**1.30**	**2.30**	**2.30**	**2.30**	**1.40**	**2.47**
2.50	**2.56**	**2.57**	**2.58**	**2.68**	**2.69**	**2.70**	**3.10**	**3.40**	**3.45**
3.50	**3.75**	**3.82**	**3.80**	**4.00**	**4.20**	**4.26**	**4.40**	**4.60**	**4.90**
5.40	**5.40**	**5.44**	**5.86**	**6.23**	**6.40**	**6.65**	**7.30**	**7.80**	**10.70**

12.30	**14.89**	**20.85**	**23.60**	**25.90**					

:statistics for the real data above The following table shows the

Table No. (2)

Coefficients	Value
Mean	**3.16874**
Variance	**20.0538**
Skewness	**3.31628**
Kurtosis	**14.9421**
Median	**1.7**
StandardDeviation	**4.47815**
Min	**0.10**
Max	**25.90**

The results of the real data will be analyzed using the best method, the greatest possibility (MLE) to measure the reliability of faults in electrical transformers and to show which of the two distributions is more suitable for these data to be represented later on other real data based on the good-matching test (*GoodnessofFit*) through three tests (Anderson-Darling , Cramér- von Mises and Kolmogorov-Smirnov (according to the hypothesis) for all distributions within the test below:

H_0: the data are distributed
H_1: the data are not distributed

Table No. (3)Goodnessof fit

Dist	EXP		ITL		ITLE	
TEST	Statistic	P-Value	Statistic	P-Value	Statistic	P-Value
Anderson-Darling	2.24417	0.06783	0.91372	0.405365	0.23428	0.978047
Cramer -von Mises	0.399706	0.072377	0.145636	0.403152	0.032654	0.966656
Kolmogorov-Smirnov	0.131792	0.067183	0.103807	0.24041	0.0532228	0.937438

:It is evident from the above table

- The value of the P-Value for the tests (Anderson-Darling, Cramér-von Mises and Kolmogorov-Smirnov) and for the distributions (EXP, ITL) is greater than the value of the level of significance 0.05 and thus does not reject the null hypothesis for these distributions, in the sense that it is preferable to represent these data on these distributions.
- The P-Value of the three tests and of the distribution (I.T.L.E). The P-Value value appeared greater than the P-Value of the (EXP, ITL) distributions. Thus, the (I.T.L.E) distribution is considered optimal for representing the real data of electrical loads compared to other distributionsIn the following . table, we show the results of data analysis according to the estimates of the probability density) functionf(t)) the cumulative distribution function ,F(t), the dependency functionR(t) and the risk) or failure functionh(t for the (ITLE).

In the following table, we show the results of data analysis according to the estimates of the probability density function (f(t), the cumulative distribution function (F)t, the dependency function R(t) and the risk or failure function (h(t) for the I.T.L.E) distribution.

Table No. (4)

i	ti	f(t)	F(t)	R(t)	i	ti	f(t)	F(t)	R(t)
1	0.10	0.145067	0.007735	0.992265	38	1.30	0.284412	0.389696	0.610304
2	0.10	0.145067	0.007735	0.992265	39	1.40	0.269538	0.417391	0.582609
3	0.14	0.188395	0.014429	0.985571	40	1.45	0.262240	0.430685	0.569315
4	0.23	0.262130	0.034926	0.965074	41	1.47	0.259354	0.435900	0.564100

5	0.29	0.296567	0.051738	0.948262	42	1.57	0.245244	0.461126	0.538874
6	0.32	0.310216	0.060845	0.939155	43	1.60	0.241124	0.468421	0.531579
7	0.40	0.337091	0.086822	0.913178	44	1.70	0.227802	0.491862	0.508138
8	0.40	0.337091	0.086822	0.913178	45	1.78	0.217620	0.509676	0.490324
9	0.48	0.352902	0.114485	0.885515	46	1.78	0.217620	0.509676	0.490324
10	0.52	0.357567	0.128701	0.871299	47	1.80	0.215142	0.514004	0.485996
11	0.55	0.359893	0.139465	0.860535	48	1.80	0.215142	0.514004	0.485996
12	0.61	0.361984	0.161137	0.838863	49	1.86	0.207871	0.526693	0.47307
13	0.64	0.361933	0.171998	0.828002	50	1.90	0.203160	0.534913	0.465087
14	0.69	0.360498	0.190065	0.809935	51	2.10	0.181214	0.573306	0.426694
15th	0.70	0.360032	0.193668	0.806332	52	2.10	0.181214	0.573306	0.426694
16	0.73	0.358316	0.204444	0.795556	53	1.16	0.305488	0.348402	0.651598
17	0.74	0.357646	0.208024	0.791976	54	1.28	0.287418	0.383977	0.616023
18	0.74	0.357646	0.208024	0.791976	55	1.30	0.284412	0.389696	0.610304
19	0.75	0.356929	0.211597	0.788403	56	2.30	0.161828	0.607569	0.392431
20	0.76	0.356169	0.215162	0.784838	57	2.30	0.161828	0.607569	0.392431
21	0.77	0.355367	0.218720	0.781280	58	2.30	0.161828	0.607569	0.392431
22	0.79	0.353643	0.225810	0.774190	59	1.40	0.269538	0.417391	0.582609
23	0.80	0.352725	0.229342	0.770658	60	2.47	0.147194	0.633813	0.366187
24	0.80	0.352725	0.229342	0.770658	61	2.50	0.144775	0.638193	0.361807
25	0.87	0.345397	0.253785	0.746215	62	2.56	0.140075	0.646737	0.353263
26	0.89	0.343055	0.260670	0.739330	63	2.57	0.139310	0.648134	0.351866
27	0.94	0.336818	0.277669	0.722331	64	2.58	0.138549	0.649524	0.350476
28	0.98	0.331504	0.291036	0.708964	65	2.68	0.131210	0.66308	0.336992
29	1.00	0.328760	0.297639	0.702361	66	2.69	0.130502	0.664316	0.335684
30	1.00	0.328760	0.297639	0.702361	67	2.70	0.129799	0.665618	0.334382
31	1.00	0.328760	0.297639	0.702361	68	3.10	0.105092	0.712383	0.287617
32	1.10	0.314412	0.329805	0.670195	69	3.40	0.090296	0.741622	0.258378
33	1.10	0.314412	0.329805	0.670195	70	3.45	0.088089	0.746081	0.253919
34	1.20	0.299476	0.360502	0.639498	71	3.50	0.085950	0.750432	0.249568
35	1.20	0.299476	0.360502	0.639498	72	3.75	0.076185	0.770667	0.229333
36	1.20	0.299476	0.360502	0.639498	73	3.82	0.073705	0.775913	0.224087
37	1.20	0.299476	0.360502	0.639498	74	3.80	0.074403	0.774432	0.225568

i	ti	f(t)	F(t)	R(t)
75	4.00	0.067785	0.788638	0.211362
76	4.20	0.061900	0.801595	0.198405
77	4.26	0.060262	0.805259	0.194741
78	4.40	0.056653	0.813440	0.186560

79	4.60	0.051964	0.824293	0.175707
80	4.90	0.045826	0.838937	0.161063
81	5.40	0.037530	0.859686	0.140314
82	5.40	0.037530	0.859686	0.140314
83	5.44	0.036954	0.861176	0.138824
84	5.86	0.031542	0.875521	0.124479
85	6.23	0.027600	0.886441	0.113559
86	6.40	0.026004	0.890996	0.109004
87	6.65	0.023868	0.897225	0.102775
88	7.30	0.019293	0.911183	0.088817
89	7.80	0.016528	0.920115	0.079885
90	10.70	0.007617	0.952959	0.047041
91	12.30	0.005323	0.963167	0.036833
92	14.89	0.003209	0.973914	0.026086
93	20.85	0.001272	0.986113	0.013887
94	23.60	0.000897	0.989056	0.010944
95	25.90	0.000688	0.990864	0.009136
Sum	301.03	19.283795	47.362699	47.637300
Mean	3.168	0.2029873	0.4985547	0.5014453

months, which means that the transformers with a capacity of 400 KVA can remain within the geographical area for a period of three months and 17 days out of 47 months, and this percentage is considered very weak, meaning that we can count on the transformer at 50% per month.

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We note from the above table:

- The dependency function was represented in the fifth column, where its value decreases with time (inverse proportion) and this is in line with the statistical theory, meaning that (the longer the period of operation of the transformer, the less its dependency), and the average dependency function reached (0.5014453), meaning that the dependence of the transformer is close to 50% per month.
- Since the cumulative distribution function is complementary to the dependency function, we note that its values are increasing with time (directly proportional) and the average of the cumulative distribution function is (0.4985547) that the transformed count is approximately 50%.
- The average operation of the transformer until the failure amounted to (3.168)

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