Mathematical Model of an Electric Hater Based on a Nano-modified Elastomer with the Effect of Temperature Self-regulation

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***Abstract*—A method for mathematical modeling of heat release in functional materials based on fluoroplastics modified with carbon nanotubes has been developed. To efficiently simulate the processes of heat release in a polymer matrix, a partial differential equation (Poisson's equation) is used, which, with physical interpretation, corresponds to the relationship of the potential field with heat release. The features of the polymer matrix and carbon nanotubes are taken into account. An equation for the percolation of electrical conductivity in a polymer is presented upon the introduction of various concentrations of carbon nanotubes. To assess the dynamics of heat release, a system of differential equations was used, which takes into account the peculiarities of heat transfer. The solution of the system of differential equations is carried out on the basis of the numerical Runge-Kutta method.**

***Keywords*—mathematical model, self-regulation of temperature, electric heater, elastomer, carbon nanotubes.**

I. INTRODUCTION

An increase in the efficiency of various technical systems using heating elements is associated with the development of new types of materials.

Nowadays, more and more new structural and functional materials appear in modern technology, which surpass traditional materials in their strength and other properties.

The most promising are filled polymeric materials (for example, fluoroplastic). The possibilities of these materials are wide enough due to the variety of polymers and fillers, as well as the methods of their modification.

The use of nanomaterials allows the implementation of new functional properties. Self-regulating electric heating can be new functional properties. The use of self-regulating electric heaters significantly increases the energy efficiency of various technologies. Self-regulating electric heaters can be used to heat internal combustion engines at low temperatures when starting after a long stay.

Another area where heaters can be used are heating systems [1].

The use of polymer materials in electric heaters allows you to achieve good results: high efficiency, system efficiency, improved operating speed, environmental friendliness and numerous additional advantages [2].

Among the wide variety of electric heaters, heaters developed using carbon nanotubes (CNTs) are of the greatest interest [3, 4] possessing the effect of temperature self-regulation due to the presence of a positive temperature coefficient of resistance [3].

Polymers modified with CNTs find industrial application in thermal interfaces [5]. The explanation of the efficiency of CNTs for polymers is based on the effect of tunneling current [6]. One of the options for heaters based on CNTs are films [7]. The properties of CNTs in a polymer matrix are influenced by the morphological features of CNTs. In this case, to increase the efficiency of CNTs in polymer matrices, binary fillers [8] and mechanical activation [9] can be used. The polymer matrix, which can be based on polyurethane [10] or an organosilicon compound [11], is of great importance for the properties of heaters.

It is worth considering the need to study the properties of temperature self-regulation. An effective approach that will justify the use of heaters with the effect of self-regulation of temperature for various technological processes is mathematical modeling [12].

The aim of the study is to develop a mathematical model of heaters with the effect of self-regulation of temperature, taking into account the external conditions of heat transfer

Objectives researches are:

1) Development of methods for mathematical modeling of heaters with the effect of self-regulation of temperature

2) Development of a mathematical model of a heater with the effect of self-regulation of temperature

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II. Materials and Methods

The introduction of an elastomer (rubber) into the fluoroplastic makes it possible to give flexibility to the composite matrix.

Modeling a blended composite, which consists of two different polymers, makes it possible to evaluate the mechanisms and patterns of optimization of mass concentrations. Combination of various properties of composites

The mass concentration of the components of a composite can be expressed:

$$C_{MT} = \frac{G_T}{G_T+G_K}, \qquad C_{MK} = \frac{G_T}{G_T+G_K} \tag{1}$$

where G is the mass of components, indices "m" and "k" correspond to elastomer and fluoroplastic.

Taking into account the density values - ρ_m and $\rho_к$, the volume concentration of Cok components is calculated:

$$C_{OK} = \frac{\rho+/\rho_K}{1/C_{MK+\rho+/\rho R-1}} \tag{2}$$

$$C_{om} = 1 - C_{ок} \tag{3}$$

$$C_{MK} = \frac{C_{ок}}{C_{ок}\left(1-\frac{\rho_T}{\rho_K}\right)+\frac{\rho_T}{\rho_K}} \tag{4}$$

The size of the fluoroplastic granules and its concentration in the composite determine the size of the unit cell. Knowing the preliminary parameters of the composite, it is possible to plan the concentration of dispersed conducting fillers.

Model of the electrical conductivity of a fluoroplastic modified with CNTs:

The percolation model of the conductivity of a polymer composite with CNTs can have the form [13, 14]:

$$\sigma = \sigma_c + (\sigma_m - \sigma_c)[(\varphi - \varphi_c)/(F - \varphi_c)]^t \tag{5}$$

σ_m — conductivity of the composite at the maximum mass content of the filler (S / cm),

σ — composite conductivity (S / cm),

σ_c — composite conductivity at the percolation threshold (S / cm),

F — maximum volume fraction of filler,

ϕ — volume fraction of filler,

ϕ_c — volume fraction of the filler at the percolation threshold,t — critical conductivity

The critical conductivity index can take values from 1 to 3.

The role of interfacial adhesion in the conductivity of a fluoroplastic modified with CNTs can be described by the expression [15]:

$$L_C = \frac{\sigma_f D}{2k} = \frac{\sigma_f R}{k} \tag{6}$$

where L_C is the minimum CNT length, which determines the maximum electrical conductivity of the CNT to the neighboring region of the polymer (matrix); D and R are the diameter and radius of CNTs, respectively; k - denotes interfacial conductivity.

It should be taken into account that a significant length of CNTs in agglomerates has waviness in nanocomposites, which reduces their efficiency, while l_{eq} is considered in the waviness parameter as:

$$u = \frac{l}{l_{eq}} \tag{7}$$

where l_{eq} is the equivalent length, i.e., the minimum distance between the two ends of each CNT.

where u = 1 corresponds to a straight CNT (without waviness), but longer u exhibits more waviness.

Waviness degrades the conductivity of a CNT, therefore, the conductivity of a wavy CNT is expressed:

$$\sigma_{CNT} = \frac{\sigma_f}{u} \tag{8}$$

Simulation of the temperature field

For a CNT-based heating element, the temperature distribution is in a functional relationship with the electric potential

$$T(x,y,z)=f(\varphi(x,y,z)). \tag{9}$$

For heaters with dimensions a, b, c along the x, y, z axes are maintained at a potential equal to 0, with the exception of the z = c face, on which the potential value is set V (x,y).

Partial differential equation in Cartesian coordinates (Poisson's equation):

$$\nabla^2\varphi = \frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} + \frac{\partial^2\varphi}{\partial z^2} \tag{10}$$

The electric potential has the form:ϕ = X(x); Y (y); Z(z). Dividing (10) by ϕ:

$$\frac{1}{X}\frac{d^2X}{dx^2} = -\alpha^2;\ \frac{1}{Y}\frac{d^2Y}{dy^2} = -\beta^2;\ \frac{1}{Z}\frac{d^2Z}{dz^2} = \gamma^2, X(x) = \sin\alpha x;$$

$$Y(y) = \sin\beta y;\ \ Z = sh\left(\sqrt{\alpha^2+\beta^2}\right). \tag{11}$$

It follows from the condition $\phi = 0$ for x = a and y = b that $\alpha a = n\pi$ and $\beta b = m\pi$, where n, m are any integers ± 1, ± 2, ± 3 ....

$$\alpha_n = n\pi/a,\ \beta_m = m\pi/b,\ \gamma_{nm} = \pi\sqrt{\left(\frac{n}{a}\right)^2 + \left(\frac{m}{b}\right)^2}, \tag{12}$$

The solution looks like: $\varphi_{nm}(x,y,z) = sin\alpha_n x \cdot sin\beta_m y \cdot sh\gamma_{nm} z$.

The solution satisfies the condition that the supply electrode is located on two sides of the heater, except for z = c. Let us expand the potential in a series in terms of the functions ϕ_{nm}:

$$\varphi(x,y,c) = V(x,y) = \sum_{n,m} A_{nm}\varphi_{nm}(x,y,c). \tag{13}$$

Expansion coefficients are determined from the boundary condition: $\varphi(x,y,c) = V(x,y) = \sum_{n,m} A_{nm}\varphi_{nm}(x,y,c)$.

The expansion in a double Fourier series has the form:

$$f(x) = \frac{A_0}{2} + \sum_{m=1}\infty\left[A_m cos\left(\frac{2\pi mx}{a}\right) + B_m cos\left(\frac{2\pi mx}{a}\right)\right] \tag{14}$$

Where $A_m = \frac{2}{a}\int_{-a/2}^{a/2} f(x)cos\left(\frac{2\pi mx}{a}\right)dx;$

$$B_m = \frac{2}{a}\int_{-a/2}^{a/2} f(x)sin\left(\frac{2\pi mx}{a}\right)dx.$$

A_{nm} can be found from the expression:

$$A_{nm} = \frac{4}{a^2 sh\gamma_{nm}c}\int_0^a dx \int_0^b dy\, V(x,y) \sin\alpha_n x \sin\beta_m y.$$

Functional dependence of the form:

$$\varphi(x,y,c) = V(x,y) = \sum_{n,m} A_{nm}\varphi_{nm}(x,y,c). \quad (15)$$

Allows to assess the distribution of the potential field inside the heater and to reveal the effective mode of the heater at various concentrations of conductive fillers - CNTs.

Development of a mathematical model of an electric heater based on a functional material

The electric heater is made according to the concept described in [11]. The heater is based on a polymer composite with CNTs.The system of differential equations for an electric heater based on a functional material has the form (for example, heating the oil sump of an internal combustion engine (Fig. 1)):

$$\rho_{\text{эн}}, C_{\text{эн}}\, h_{\text{эн}}\, F_{\text{эн}}\frac{T_h}{d\tau}=(\mathrm{P}(T_{\mathrm{h}}))\text{- } K_1\, F_{\text{к}}\,(T_{\mathrm{h}}\text{- } T_{\text{к}}) \quad (16)$$

$$\rho_{\mathrm{k}}, C_{\mathrm{k}}\, h_{\mathrm{k}}\, F_{\mathrm{k}}\frac{T_o}{d\tau}= K_1\, F_{\text{к}}\,(T_{\mathrm{h}}\text{- } T_{\text{к}})\text{- } K_2F_{\mathrm{o}}\,(T_{\text{к}} - T_{\mathrm{o}})\text{- } K_3F_{\mathrm{ou}}\,(T_{\mathrm{o}}\text{ - } T_{\text{к}})(17)$$

Where F_{h}, $F_{\text{к}}$, F_{ou},– areas of the electric heater and oil sump, the total area of the heater and crankcase in contact with the environment, м^2; P (T_{h}) – electric power of the heater, Вт; T_{h}, $T_{\text{к}}$, T_{o}, T_{ou} – temperature of heater, crankcase, engine oil and environment, °C; C_{h}, $C_{\text{к}}$, C_o – heat capacity of electric heater, crankcase and engine oil, Дж/(кг·°С); K_1 , K_2, K_3 – heat transfer coefficients from the electric heater to the crankcase, from the crankcase to the engine oil, from the crankcase and heater to the environmentВт/(м^2·°С); ρ_{h}, $\rho_{\text{к}}$, ρ_{o} – density of the heater, crankcase and engine oil material, kg / m^3; h_{h}, $h_{\text{к}}$ – heater and crater height, м; τ – time, с.

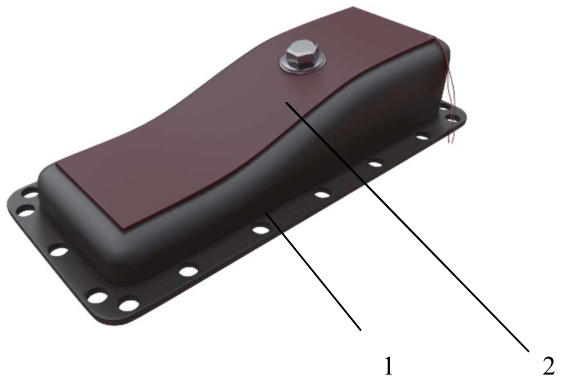

Fig.1. Heater (2) on the crankcase of the combustion engine (1)

The power of the self-regulating heater has a functional dependence on temperature:

$$P(T_{эн}) = 5 - 20\times10^{-2}T_{эн} + 7\times10^{-3}T_{эн}^{\,2} - 2\times10^{-5}T_{эн}^{\,3} \quad (18)$$

III. RESULTS

A sample of a heater based on a nanomodified elastomer has the form shown in Figure 2.

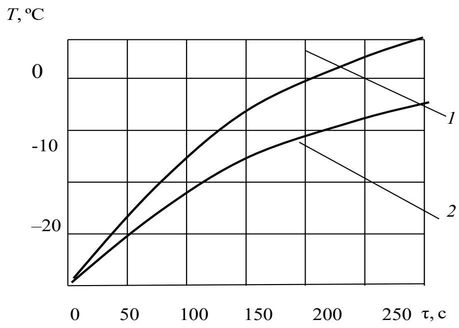

Fig.2. Dynamics of temperature change of heater 2 and engine oil 1.

IV. DISCUSSION

When solving the system of differential equations, the Runge-Kutta method was used. Figure 2 shows how the temperature of the engine oil in a heated unit changes over time.

V. CONCLUSION

A method for mathematical modeling of heat release in functional materials based on fluoroplastics modified with carbon nanotubes has been developed. This technique takes into account the formation of percolation in the polymer and the features of dissimilar polymer materials, as well as the relationship of the potential field with heat release.

A mathematical model of heat release in functional materials based on fluoroplastics modified with carbon nanotubes, based on the equation of mathematical physics, has been developed. to obtain the modes of heat release of the heater.

A mathematical model of an electric heater based on a functional material based on a system of differential equations has been developed, the solution of which by the Runge-Kutta method allows determining the dynamics of changes in the temperature regime of the heater and the heated object.

REFERENCES

[1] Jérémie Léger, Daniel R. Rousse, Kilian Le Borgne, Stéphane Lassue, Comparing electric heating systems at equal thermal comfort: An experimental investigation, Building and Environment, Volume 128, 2018, pp. 161-169, ISSN 0360-1323, https://doi.org/10.1016/j.buildenv.2017.11.035.

[2] Yi Liu, Eric Asare, Harshit Porwal, Ettore Barbieri, Stergios Goutianos, Jamie Evans, Mark Newton, James J.C. Busfield, Ton Peijs, Han Zhang, Emiliano Bilotti. The effect of conductive network on positive temperature coefficient behaviour in conductive polymer composites. Composites Part A 139 (2020) 106074

[3] Yanpei Fei, Feng Chen, Wei Fang, Lixin Xu, Shilun Ruan, Xianhu Liu, Mingqiang Zhong, Tairong Kuang. High-strength, flexible and cycling-stable piezo-resistive polymeric foams derived from thermoplastic polyurethane and multi-wall carbon nanotubes. Composites Part B199 (2020), 108279

[4] Kunmo Chu, Dong-Jin Yun, Dongouk Kim, Hyokeun Park, Sung-Hoon Park. Study of electric heating effects on carbon nanotube polymer composites. Organic Electronics 15 (2014) 2734-2741

[5] Er-Chieh Cho, Cai-Wan Chang-Jian, Yu-Sheng Hsiao, Kuen-Chan Lee, Jen-Hsien Huang. Three-dimensional carbon nanotube based polymer composites for thermal management. Composites: Part A 90 (2016) 678-686

[6] Wei Gao, Zhonggang Zhang, Yu Zhang, Binghe Ma, Jian Luo, Jinjun Deng, Weizheng Yuan. Efficient carbon nanotube/polyimide composites exhibiting tunable temperature coefficient of resistance for multi-role thermal films. Composites Science and Technology 199 (2020) 108333

[7] Tetsuya Yamamoto, Kan Kawaguchi. Synthesis and Electrical Properties of Composite Films Comprising Polymer Particles and Carbon Nanotubes. Colloid and Interface Science Communications 20 (2017) 5-8

[8] Shchegolkov A.V. The comparative analysis of thermal effects in elastomers modified with MCNT at constant DC voltage. Science Vector of Togliatti State University. 2021;(1): pp. 63-73. (In Russ.) https://doi.org/10.18323/2073-5073-2021-1-63-73

[9] Shchegolkov A.V. Multistage Mechanical Activation of MWCNTS to Improve Percolation Transitions in the Elastomer / MWCNTS System: Approaches to the Implementation and Practice of Modifying Elastomers. Vestnik Magnitogorskogo Gosudarstvennogo Tekhnicheskogo Universiteta im. G.I. Nosova [Vestnik of Nosov Magnitogorsk State Technical University]. 2021, vol. 19, no. 2, pp. 58–67. https://doi.org/10.18503/1995-2732-2021-19-2-58-67

[10] Luo J., Lu H., Zhang Q., Yao Y., Chen M., Li Q. Flexible carbon nanotube/polyurethane electrothermal films // Carbon. 2016. vol. 110. pp. 343–349. DOI: 10.1016/j.carbon.2016.09.016.

[11] Ali, I., AlGarni T.S., Shchegolkov A., Shchegolkov A., Jang S.-H., Galunin E., Komarov F., Borovskikh P., Imanova G.T. Temperature self-regulating flat electric heaters based on MWCNTs-modified polymers // Polymer Bulletin. 2021. Article in press. DOI: 10.1007/s00289-020-03483-y.

[12] Pitiya Kamonpatana, Hussein M.H. Mohamed, Mykola Shynkaryk, Brian Heskitt, Ahmed E. Yousef, Sudhir K. Sastry, Mathematical modeling and microbiological verification of ohmic heating of a solid–liquid mixture in a continuous flow ohmic heater system with electric field perpendicular to flow, Journal of Food Engineering,Volume 118, Issue 3, 2013, pp. 312-325,https://doi.org/10.1016/j.jfoodeng.2013.04.005.10.

[13] Mamunya E. P., Davidenko V. V., Lebedev E. V. Percolation conductivity of polymer composites filled with dispersed conductive filler. Polymer composites. – 1995. – Т. 16. – №. 4. – С. 319-324.

[14] 11. Mamunya, Y. P., Davydenko, V. V., Pissis, P., & Lebedev, E. V. (2002). Electrical and thermal conductivity of polymers filled with metal powders. European Polymer Journal, 38(9), 1887–1897. doi:10.1016/s0014-3057(02)00064

[15] Zare, Y., Rhee, K.Y. Effect of interfacial/interphase conductivity on the electrical conductivity of polymer carbon nanotubes nanocomposites. Engineering with Computers (2020). https://doi.org/10.1007/s00366-020-01062-3