A Study on The Determinant Spectrum and Performance of STTC on Slow Fading Channels

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ABSTRACT

Wireless communication has faced fading which is one of the major problems. Fading is a result of multipath signals, multi-input multi-output MIMO system is used to minimize the effect of this phenomenon. MIMO is meant to be more than one copy of the same information that is sent by more than one transmitter antenna. Space-Time Trellis Codes (STTCs) are a technique which can be used to enhance the performance of wireless communications systems over fading channels. It is combination of space and time diversity. It provides capacity benefits in fading channels, and helps to improve the reliability and the data rate of wireless communication. Several researchers have undertaken the construction of Space-Time Trellis Codes. The Rank and Determinant Criteria (R&DC) and Euclidean Distance Criteria (EDC) have been developed as design criteria.

In this thesis, the effect of determinant spectrum on code design and performance of Space-Time Trellis codes (STTCs) are discussed. Some new 4-state and 8-state 4-PSK and 8-state 8-PSK are constructed as well. The determinant spectrum and the simulation results indicate that the new constructed codes are superior in performance to some existing STTC schemes for a small number of independent subchannels. It is also important to note that many error events and not only the first one are dominating the performance of STTC on a slow fading channel.

Keywords: STTCs, Determinant spectrum, Error event, Distance matrix.

ÖZ

kablosuz iletişim önemli sorundan biri olan solma ile karşı gelmiştir. Bu olayın etkisini düşürmek için MIMO sistemi kullanılmıştı MIMO, aynı bilginin birden fazla kopyasının birden fazla verici antenle yollanması anlamına gerlir. STTC, kablosuz iletişimin, solmakta olan kanallar üzerine performansını gelistırmek için kullanılan bir tekniktır. Bu, uzay ve zaman çeşitliğinin kombinasyonudur, solmakta olan kanallarda kapasite faydalar sağlar ve kablosuz iletişimin emniyetini ve veri hızını geliştırır. çesitlı araştırmacılar STTS nin inşaatını üstlenmişlerdir. R&DC ve EDC tasarım krıterleri olarak tasarlanmıştır

Bu tezde, belırleyici spektrumun kod tasarımı üzerindeki etkisi ve STTC nin performansı tartışılmıştır. Bazı yeni 4-state ve 8-state 4-psk ve 8-state 8-psk kurumuştur. Belirleyici spekturum ve simulasyon sonuçları, yeni kurulan kodların, az sayıda bağımısız alt kanal nedeniyle bazı mevcut STTC düzenlerin güre daha üstün performansı olduğ unu belirtir. Birçok hata olayları ve ilk sadece yavaş bir solma kanalda STTC performansını hakim olduğunu da not etmek önemlidir.

Anahtar Kelimeler: STTCs, Determinant spektrum, hata olayı, Uzaklık matrisi.

To my parents who helped and encouraged me and they help me to reach to this stage

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LIST OF ABBREVIATIONS

1G	First Generation
2G	Second Generation
3G	Third Generation
4G	Fourth Generation
AWGN	Additive White Gaussian Noise Channel
Bs	Signal Bandwidth
BW	Bandwidth
Bc	Coherent the Bandwidth
dB	Decibel
CSI	Channel State Information
CID	Connection Identifier
CINR	Carrier to Interference to Noise Ratio
CNR	Carrier to Noise Ratio
DL	Downlink
DSL	Digital Subscriber Line
EDC	Euclidean Distance Criteria
FDMA	Frequency Division Multiple Access
GSM	Global System for Mobile communication
LAN	Local Area Network
LOS	Line Of Sight
LTE	Long Term Evolution
ML	Maximum Likelihood
MIMO	Multi Input Multi Output

Nt	Number of Transmit Antenna
Nr	Number of Receive Antenna
NLOS	Non Line Of Sight
PSK	Phase Shift Key
PEP	Pair-wise Probability of Error
QoS	Quality of Service
ST	Space-Time
STC	Space-Time Code
SISO	Single-Input/Single-Output
STTC	Space-Time Trellis Code
SNR	Signal to Noise Ratio
SMS	Short Messaging Service
STBCs	Space–Time Block Codes
Tc	Coherence Time
Ts	Signal Period
TDMA	Time Division Multiple Access

Chapter1

INTRODUCTION

1.1 Background

Wireless communication has faced many problems such as fading since its first development in 1897. It is one of the industries that have grown rapidly by increasing throughput through wireless channels, and increasing reliability of the wireless communication. The main motivations behind the fast development of wireless communication are the portability, accessibility, and mobility [1]. In other words, freedom is offered by wireless, from being limited to a specific location of a fixed environment. This freedom is considered to be the major driving force for users; the punishment for this freedom is often lower quality, risk of disconnection, or lower throughput in comparison with the equivalent wired communication [2].

Wired communication is extremely reliable and more balanced, but its area is limited. Therefore, the wireless communication is better choice for users.

A difficult task of limited availability of radio frequency spectrum and complex time varying problems in the wireless channel were faced by wireless designers. Multipath and fading, as well as meeting the demand for high data rates are examples of these problems. Simultaneously, there is an urgent need for better quality of service (QoS). In the last century, the wireless communication testified the noticeable development and became more used than wired; however the request for the bandwidth (BW) and the capacity was increased. Then the Space-Time Code (STC) was introduced by Tarokh et al [3] in 1998 to combat the fading, increase the BW, and improve the capacity.

1.2 Wireless Applications

There are several systems in which wireless communication is applied. Radio broadcasting is one of the earliest successful joint applications, such as Television broadcasting and satellite communications. The first generation (1G) has presented the fundamentals of the cellular phones at the early 1980s. Cellular telephone system is also referred to as Personal Communications System (PCS), which is very famous and profitable worldwide system. This system is aimed to provide two-way of voice communication. 1G used Frequency Division Multiple Access (FDMA). In 1990's, the second generation (2G) technology has presented to use digital techniques and signal processing techniques. 2G supplies the basic of voice service and Short Messaging Service (SMS). Global System for Mobile communication (GSM) network was introduced at this time. GSM uses Time Division Multiple Access (TDMA). Third generation 3G has followed the 2G which has looked for information at high-speed. WiMAX was introduced at this period. Fourth Generation (4G) is a stretch of 3G, and provides broad range of the most multimedia applications .The main advantage of 4G is, that it is a complete IP-based network and it provides high data rate which helps us to access to Wireless Local Area Networks.

1.3 Wireless Channels

In 1864, James Clerk Maxwell formulated a theory of electromagnetic propagation which prophesied the presence of radio waves. This theory can be represented to be the basic of wireless communication field [4]. Wireless channels work by electromagnetic radiation from the transmitter to the receiver. Electromagnetic waves broadcast through the medium where they are scattered, reflected, and diffracted by buildings, terrain, walls, and other objects as shown in Figure 1.1.

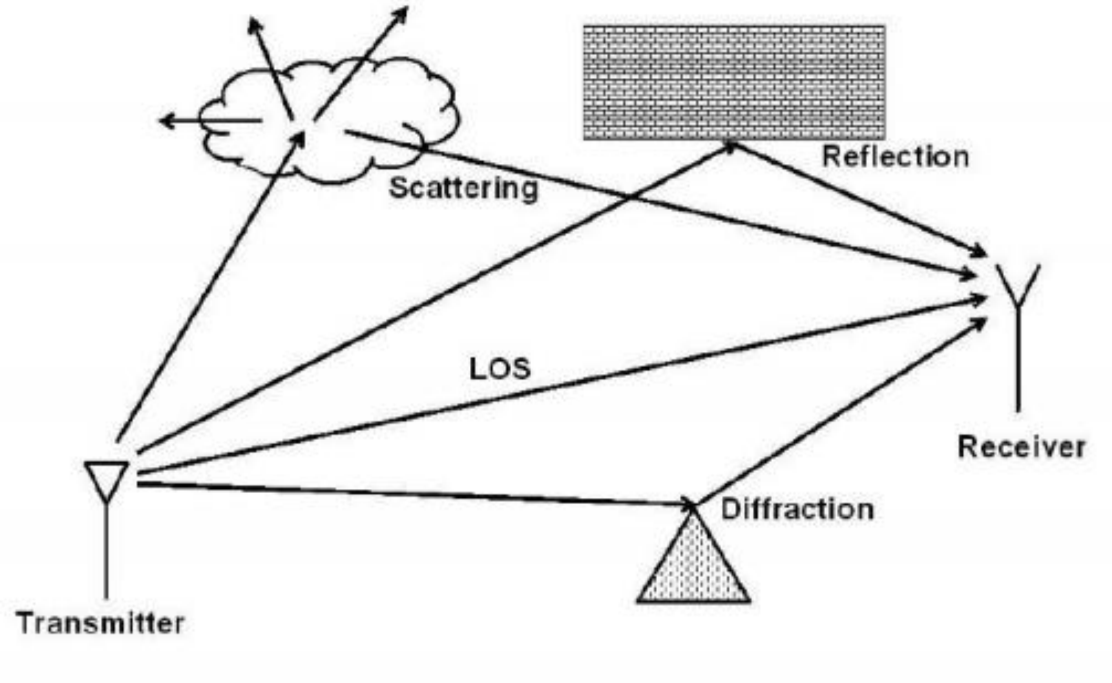

Figure 1.1: Different Path in Wireless Channel

There are several paths between the transmitter and the receiver, and then different versions of the transmitted signal are received at the receiver, all accumulated signals together generate a non-additive model for the wireless channel “additive white Gaussian noise channel” (AWGN) [1]. That means the characteristic of the wireless channel which is the difference of the channel strength over time and frequency. These differences are classified into two types:

- Large-scale fading or Attenuation.
- Small-scale fading or Fading.

Attenuation: Generally, attenuation means the loss of the signal's amplitude with increasing propagation distance. The factors are causing attenuation including propagation losses, antenna losses and filter losses [5].

Multipath fading occurs in when there are more than two transmitted signals with different amplitude and phases that are additively combined at the receiver [6].

Fading can also be characterized based on the frequency dispersion parameters as:

- Slow fading or a long-term: it appears when there are large reflection and diffraction objects along the transport path and the moving of the terminal to this distance is very short and corresponding change slowly for propagation waves [5].
- Fast fading or short-term: it is component related with multipath propagation [7]. Fast fluctuation of amplitude occurs when the terminal moves short distance. It is affected by the quickness of the mobile terminal and the transition BW of the signal.

The fading channels can be classified based on their multipath time delay into flat and frequency selective and based on Doppler spread into slow and fast. These two phenomena are independent of each other and result in the following four types of fading channels [8]:

- Flat Slow Fading or Frequency Non-Selective Slow Fading: this type occurs in when the coherence the BW (B_c) of the channel is larger than the BW of the signal (B_s), and the coherence time (T_c) of the channel is larger than the signal period T_s.

- Flat Fast Fading or Frequency Non-Selective Fast Fading: this type occurs when the coherence BW of the channel is larger than the BW of the signal, and the coherence time of the channel is smaller than the signal period.
- Frequency Selective Slow Fading: this type occurs when the coherence BW of the channel is smaller than the BW of the signal, and the coherence time of the channel is larger than the signal period.
- Frequency Selective Fast Fading: this type occurs when the coherence BW of the channel is smaller than the BW of the signal, and the coherence time of the channel is smaller than the signal period. Table 1.1 shows fading types and their characteristics.

Table 1.1: Fading Types and Characteristics

Type of fading channel	Characteristics
Frequency non-selective (flat) slow fading	$B_s \ll B_c$ $T_s \ll T_c$
Frequency non-selective (flat) fast fading	$B_s \ll B_c$ $T_s \gg T_c$
Frequency-selective slow fading	$B_s \gg B_c$ $T_s \ll T_c$
Frequency-selective fast fading	$B_s \gg B_c$ $T_s \gg T_c$

1.3.1 Additive White Gaussian Noise (AWGN) Channel Model

(AWGN) channel model is as shown in Figure 1.2 It is a basic noise model applied in information theory to simulate the impact of many random operations that happen in nature. It is a good model for a lot of satellite and profound space communication links [1]. Because of multipath, it is bad model for most earthly links [9]. In this model, the received signal can be expressed as:

$$r(t) = x(t) + \eta(t) \qquad 1.1$$

where, *x(t)* is the transmitted signal at *t* time and $\eta(t)$ is the noise exemplified as a sample function from a Gaussian random process with zero mean and variance N_0. The noise *n (t)* is assumed to be independent of the signal *x (t).*

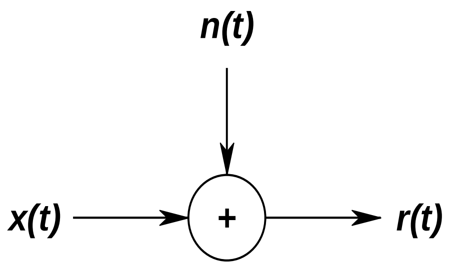

Figure 1.2: (AWGN) Channel Model

1.3.2 Rayleigh Fading Channel Model

This type of channel model is usually used in wireless communication systems. It is used when there is non- line of sight LOS among the transmitter and the receiver [10]. This type supposes that the size of the received signal fade depends on the Rayleigh distribution. The received signal is corrupted by multipath fading as well as AWGN. More than one version of the transmitted signal is received as a result of multipath propagation. Each version has a distinct delay time, Doppler shift, attenuation and phase [6]. When flat or frequency non-selective fading exists, the received signal in complex baseband form can be expressed as:

$$r(t) = h(t)x(t) + \eta(t) \qquad 1.2$$

where, *x(t)* is the signal transmitted and *h(t)* is the channel state information (CSI).

1.4 Diversity

The channel is fading when the signal power fluctuates in wireless channel and it drops to a large extent. The wireless channel uses the diversity in order to anti fade, which reduces the effect of multipath fading, this leads to improve the reliability of transmission [11].

In wireless channels, a non-realistic solution to reduce the effect of fading may be to increase the height of antenna or it is size and transmission power. A practical alternative to these solutions would be to transmit some replica of the signal to the receiver thereby increasing the probability that the receiver will receive a less damaged signal. This is the basic idea of diversity [12]. Many diversity techniques are available. Discuss some of them below.

1.4.1 Polarization Diversity

In polarization diversity, vertically and horizontally polarized signals are used to achieve diversity. It uses either two transmit antennas or two receive antennas with different polarization. The two transmitted waves follow the same path. However, since the multiple random reflections distribute the power nearly equally relative to both polarizations, the average receive power corresponding to either polarized antenna is approximately the same. Since the scattering angle relative to each polarization is random, it is highly improbable that signals received on the two differently polarized antennas would be simultaneously in deep fades.

Unlike spatial diversity, polarization diversity does not require separate physical locations for the antennas. However, polarization diversity can only provide a diversity order of two and not more.

1.4.2 Frequency Diversity

In frequency diversity, replicas of the information signal are transmitted from different carrier frequencies. To achieve diversity, the carrier frequencies must be separated by more than the coherence bandwidth of the channel so that the replicas of the signal experience independent fades.

Similar to temporal diversity, frequency diversity suffers from bandwidth deficiency. It also requires additional transmit power to send the signal over multiple frequency bands. Also the receiver needs to tune to different carrier frequencies [13].

1.4.3 Temporal Diversity

In temporal or time diversity, replicas of the information signal are transmitted in different time slots. To achieve diversity, two adjacent time intervals must be separated for more than the coherence time of the channel so that the replicas of the signal experience independent fades. We get multiple, uncorrelated repetitions of the signal at the receiver. Time diversity can also be achieved through coding and interleaving [13].

1.4.4 Spatial Diversity (Antenna Diversity)

This type is popular to use in wireless communication systems. Two or more separated antennas at both transmitter and receiver are used. The aim of the space diversity is to decrease the effect of multipath fading [4] .

1.4.5 Transmit diversity

Transmit diversity implements multiple antennas at the transmitting side. In this diversity scheme, controlled redundancies are introduced at the transmitter and at the receiver side; suitable signal processing technique is implemented. For this technique, channel information at the transmitter is required. But due to the use of

Space- Time Coding schemes, such as Alamouti's scheme, it has become possible to achieve transmit diversity without getting channel information. Multiple antennas used either in transmitting and receiving side increases the performance of a communication system in fading environment. In case of a mobile radio communication system, employing multiple antennas at base station is more effective but single or double antennas can be employed in mobile units. In case of transmitting from mobile to base station, the diversity can be achieved through multiple receive antennas and while transmitting from base station to the mobiles, diversity is achieved through multiple transmit antennas. Transmit diversity is gaining popularity due to its simple implementation and feasible for having multiple antennas at the base station that improves the downlink and is one of the best methods of brushing the detrimental effects in wireless communication.

1.5 Multi-Antenna Transmission Systems

Generally, the system wireless communication that composed of the transmitter, receiver and channel classified dependent to the number of antennas that used in the system.

The simplest communication system is the single-input/single-output (SISO) system which uses one transmits and one receives antennas. However, a more sophisticated communication system is the multiple-input/multiple-output (MIMO) system which uses a multiple of transmits and receives antennas.

1.5.1 MIMO Channel Model

In this model, multiple antennas are used at the transmitter and receiver to enhance communication performance. MIMO technology in wireless communications is very interesting because it provides considerable increases in data throughput and link range without extra BW or increased transfer power. This aim is achieved by spreading some total transmitting power across the antennas to obtain an array gain that enhances the spectral efficiency (more bits per second per hertz of BW) to enhance a diversity gain to decrease fading.

Suppose we have MIMO system, this system consists of: N_t which is the number of transmit antennas and N_r is the number of receive antennas.

System MIMO model as shown in Figure 1.3, the signals $c_{t,i}$, i =1,..,N_t, are simultaneously transmitted from N_t. Each transmitted signal is influenced by channel fading, and the response from each signal from each of N_t is received at eachN_r. If we consider a flat-fading channel with the channel gain between receive antenna *j* and transmit antenna *i* have given as *hi*,*j*, then the received signal at *j* at time *t* is given by.

$$r_{t,j} = \sum_{i=1}^{N_t} h_{i,j} c_{t,j} + \eta_{t,j} \qquad 1.3$$

where, $\eta_j(t)$ is AWGN noise with zero mean and variance $N_0/2$ per dimension in the receive antenna j at time.

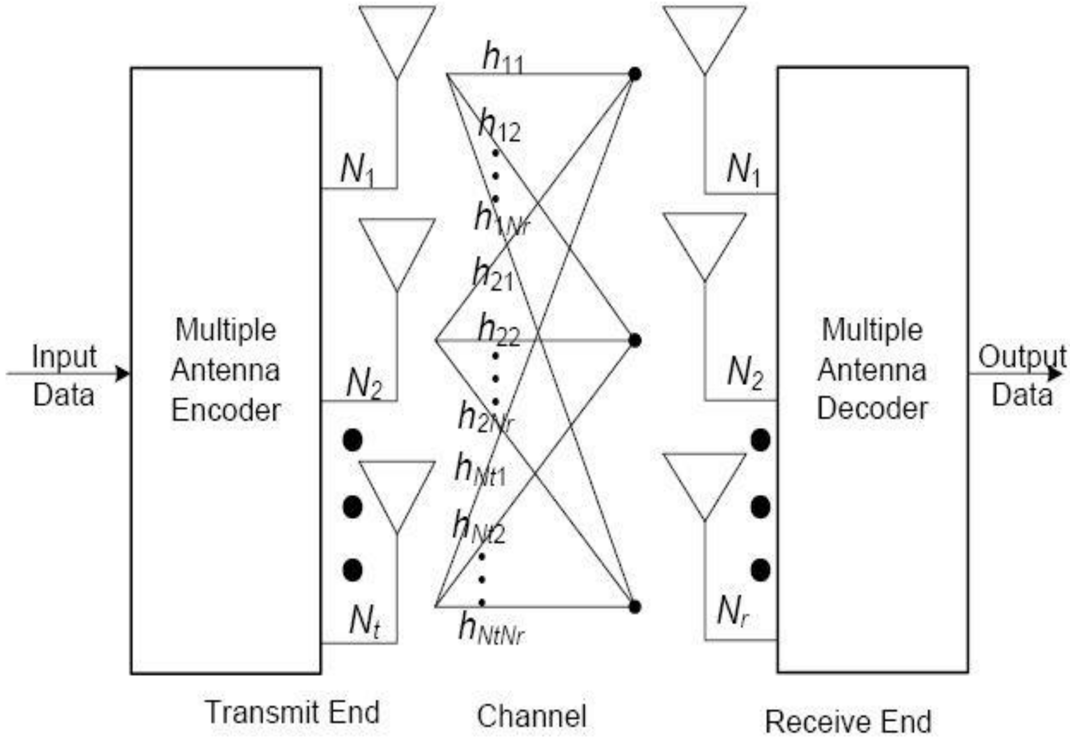

Figure 1.3: MIMO System

The received signal can be written *R*:

$$R = \begin{bmatrix} r_{1,1} & r_{1,2} & \cdots & r_{1,n_r} \\ r_{2,1} & r_{2,2} & & r_{2,n_r} \\ \vdots & & \ddots & \vdots \\ r_{n_t,1} & r_{n_t,2} & \cdots & r_{n_t,n_r} \end{bmatrix}$$

Also, the channel fading coefficients are constant during the transmission period, these fading channel coefficients may also be written matrix *H* from as:

$$H = \begin{bmatrix} h_{1,1} & h_{1,2} & \cdots & h_{1,n_r} \\ h_{2,1} & h_{2,2} & & h_{2,n_r} \\ \vdots & & \ddots & \vdots \\ h_{n_t,1} & h_{n_t,2} & \cdots & h_{n_t,n_r} \end{bmatrix}$$

1.6 MIMO Channel Capacity

The channel capacity in a MIMO system is greater than that of a (SISO) system [8]. Channel capacity is the maximum error-free data rate that a channel can back. The capacity of an AWGN channel was derived by Shannon in [14], it is as in equation below:

$$C = \log_2(1+\gamma) \qquad 1.4$$

where γ is the average received signal to noise ratio.

The capacity of a deterministic SISO channel is given by [8].

$$C = \log_2(1+\rho|H|^2) \qquad 1.5$$

where $|H|^2$ is the normalized channel power transfer characteristics and ρ is the average SNR at the receiver [8].

The channel capacity for a MIMO system is given by [8].

$$C = \log_2\left[\det(I_{N_R} + \frac{\rho}{N_t} HH^H)\right] \qquad 1.6$$

while in an ergodic MIMO channel, the capacity is given by:

$$C = E_H\left\{\log_2\left[\det(I_{N_R} + \frac{\rho}{N_t} HH^H)\right]\right\} \qquad 1.7$$

where E_H is expectation with respect to H, $I_{N_r} = [N_t * N_r]$ identity matrix and det[A] determinant of matrix **A**. when the increase number of antennas the ergodic capacity are increase [8].

1.7 Thesis Objective

In this thesis, the determinant spectrum (DS) of Space-Time Trellis Codes on slow fading channels is considered. The first five spectrum lines of the DS for some existing schemes are computed. Some new 4 and 8-states 4-PSK as well as 8-state 8-PSK codes are constructed. The performance of these new schemes has shown to be superior to those of some existing schemes.

1.8 Thesis Outline

The remainder of this thesis is organized as follows: In Chapter 2, the general information about Space-Time Trellis Codes, Space Time Code encoder, some examples on Code Constructions for 4-PSK and 8-PSK constellations are presented. In addition the rank and determinant criteria and some concepts are introduced in this chapter. Chapter 3 mentions about the performance analysis and the simulation results for the new codes and the comparison between them and some existing codes. Finally Chapter 4 includes the conclusions and future work.

Chapter 2

SPACE TIME CODES

Space-Time Coding (STC) which was introduced by Tarokh [3]in 1998; is a group of practical signal design mechanisms which provides an effective way for achieving diversity in fading channels through redundancy implementation at the transmitter across both space and time. STC is a technique which is used to enhance the reliability of data transmitted, capacity, and enhance performance in wireless communication systems by utilizing multiple transmits antennas. STCs depend on multiple transmissions that increases the data flow copies to the receiver. Tarokh [3] designed a theoretical basis on STCs; he developed the rank and determinant criteria. The rank can give full diversity; it is very useful criteria for multiple input and multiple output (MLMO) systems [7] and the determinant criterion optimizes the coding gain.

STCs can be classified into two main types:

- Space–time block codes (STBCs).
- Space–time trellis codes (STTCs).

2.1 Space–Time Block Codes (STBCs).

STBCs codes were proposed by Alamouti [15]. Alamouti first presented transmit diversity technique; for two transmit antennas, with a simplified decoding algorithm, and it extended to more than two transmit antennas. STBCs are achieving full diversity but suffer from a lack of coding gain. The signal at each receive antenna is a linear superposition of the n transmitted signals troubled by noise. Simple way achieves Maximum likelihood decoding through decoupling of the signals that are transferred from multiple antennas rather than common detection.

2.2 Space–Time Trellis Codes (STTCs)

A STTCs scheme corresponds a joint combination of a convolutional code; a modulation scheme and a set of transmit and receive antennas in order to break the effect of fading.

The main goal of STTC design is to combat the impact of fading. Also, at the same time it can progress a large coding gain, spectral efficiency, and betterment the diversity on flat fading channels [16]. STTC is a way to represent the codes in trellis form. In [3]the authors found the design criteria for STTCs over slow frequency non-selective fading channels. The design criterion depends on the rank, determinant and the trace for generated matrices which are generated from pairs of distinguished codewords. The diversity gain depends on the minimum rank of distance matrices. Figure 2.1 below shows the system model for STTC [8].

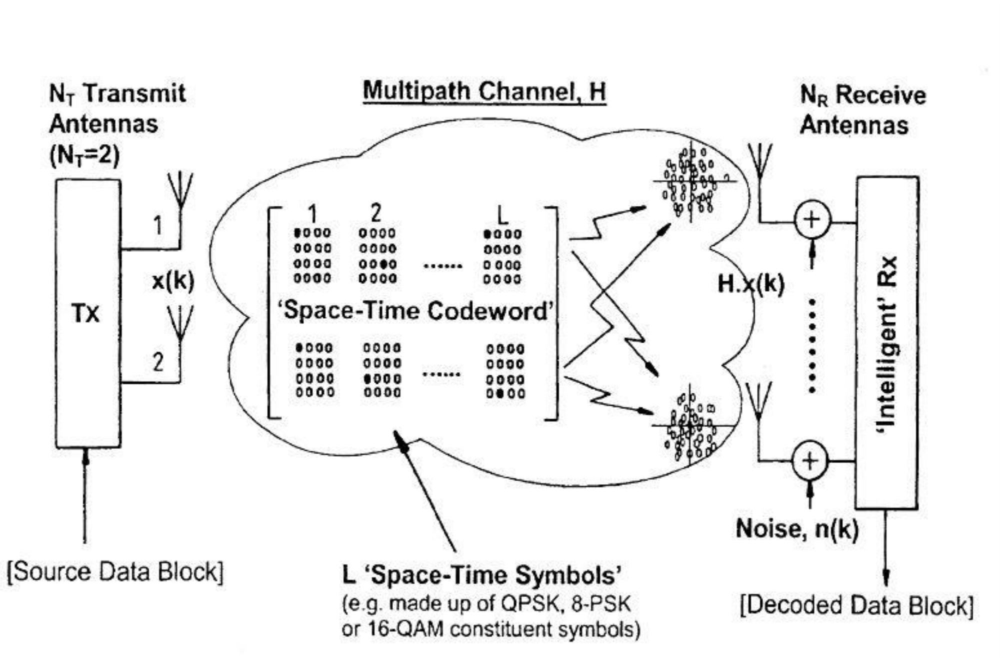

Figure 2.1: System Model

2.3 System Model

Assume the communication system shown in Figure 2.1. A STTC model based wireless system in the transmitter base station has an encoder, modulator, pulse shaper and multiple antennas as shown in Figure 2.2. In this thesis we used two transmit antennas $N_t = 2$ at the transmitter,

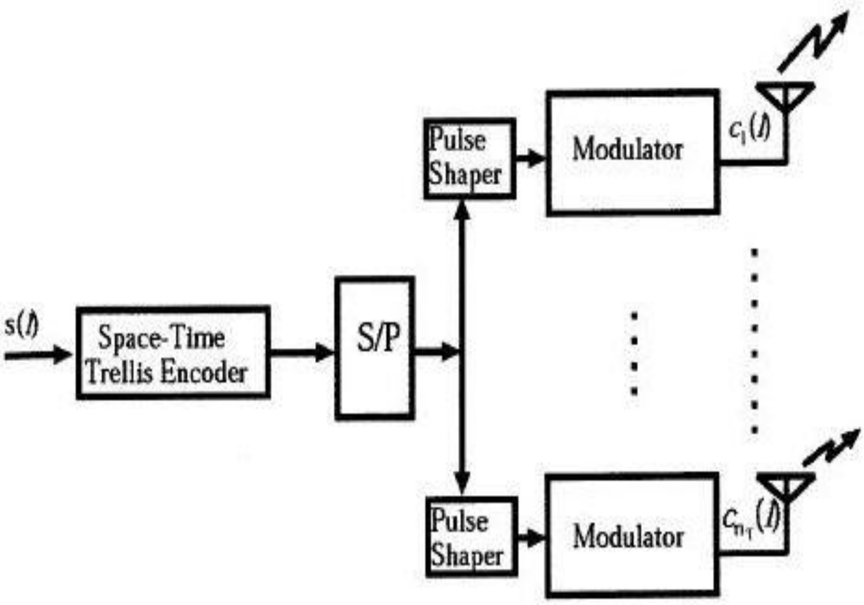

Figure 2.2: Block Diagram of Transmitter **[13]**

The receiver station has one or more receiver antenna $N_r \geq 1$, channel estimator, demodulator and S'ITC decoder as shown in Figure 2.3. In this thesis $N_r = 1$

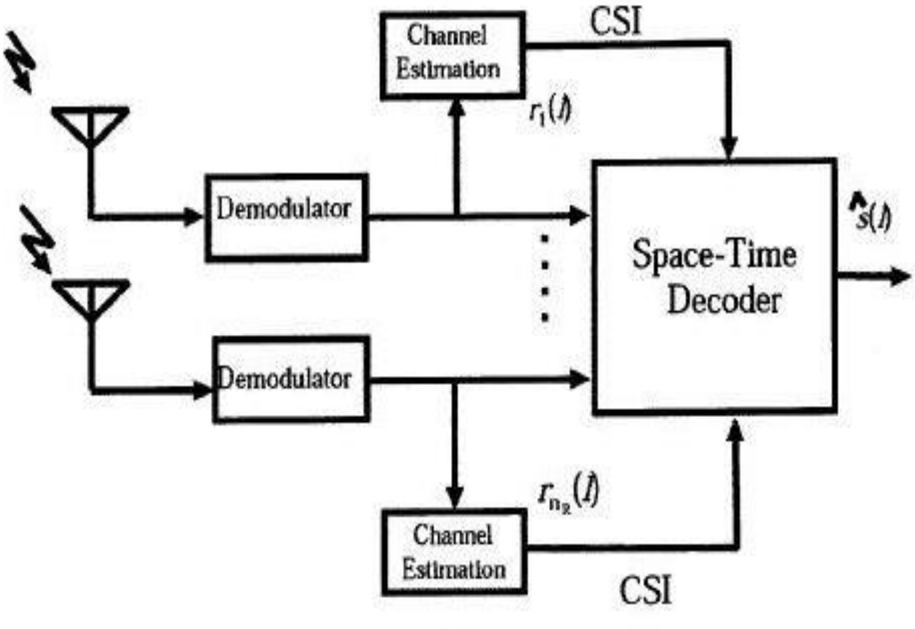

Figure 2.3: Block Diagram of Receiver **[13]**

Let c_t the information that fed the encoder. Which define $c_t = (c_t^1, c_t^2, c_t^3, \dots, c_t^m)$. Where $m = \log_2 M$ bits per symbol, M is the size of constellation. The data input encoded by STTC encoder, this data passes through a serial-to-parallel converter and is divided in to *n* streams of data which are input into a pulse shaper. Modulation is applied to the output of the pulse shaper, at each time slot *t*, the modulated output, is a signal that is transmitted by transmit antenna i $x_t = \left(x_t^1, x_t^2, x_t^3, \dots, x_t^{N_t}\right)^T$, where T means the transpose of a matrix. The transmitted symbols have energy E_s. We assume that the *n* signals are transmitted from the transmit antennas simultaneously from the transmit antennas. The signals have transmission period *T*. The vector of coded modulation symbols from different antennas x_t, is called a space-time symbol. We assume a quasi-static Rayleigh fading channel model. When the signals transmit through the medium, these signals undergo to the Rayleigh fading. So, the signal at each N_r of the receive antennas is a noisy superposition of the transmitted signals, each of which has undergone fading.

At the receiver, the demodulator computes a decision statistic based on the received signals arriving at each antenna. The signal received by antenna *j* at time *t* is given by [7]

$$r_{t,j} = \sqrt{E_s} \sum_{i=1}^{N_r} h_{i,j} c_{i,}(t) + \eta_j(t) \qquad 2.1$$

where $\eta_j(t)$ is an AWGN noise with zero mean and variance $N_0/2$ per dimension at the receive antenna *j* at time *t*. E_s is the transmitted symbol energy. The channel coefficient $h_{i,j}(t)$ in quasi-static is assumed fixed over a frame and change from frame to frame.

The received signals from N_r at time t represents as $r_t = (r_t^1 r_t^2 r_t^3 \cdots r_t^{N_r})^T$, and the noise can be describe as $\eta_t = (\eta_t^1 \eta_t^2 \eta_t^3 \cdots \eta_t^{N_r})^T$

in the receiver the decoder uses Viterbi algorithm to perform maximum likelihood (ML) decoding, to estimate the transmitted information sequence and assume the receiver has perfect channel state information (CSI). At the receiver, the decision metric is computed based on the squared Euclidean distance between the assumed received sequence and the actual received sequence as [13],

$$\sum_t \sum_{j=1}^{n_r} \left| r_t^j - \sum_{i=1}^{n_t} h_{ji}^t x_t^i \right|^2 \qquad 2.2$$

2.4 STTC Encoder

For STTCs, binary data to modulation symbols is mapped by the encoder, where a trellis diagram describes the mapping function. The M-PSK modulation with N_t transmit antennas are considered in Figure 2.4 below as an encoder of STTC, c is referred to the input message stream; it is as shown below [16]:

$$c = (c_0, c_1, c_2, \dots, c_t, \dots) \qquad 2.3$$

where c_t is a set of $m = \log_2 M$ (information bits at time t), as shown below:

$$c_t = (c_t^1, c_t^2, c_t^3, \dots, c_t^m) \qquad 2.4$$

The input sequence into an M-PSK modulated signal sequence is mapped by the encoder, which is given by:

$$x = (x_0, x_1, x_2, \dots, x_t \dots) \tag{2.5}$$

where x_t is a space-time symbol at time t, it is as shown in below:

$$x_t = \left(x_t^1, x_t^2, x_t^3, \dots, x_t^{N_t}\right)^T \tag{2.6}$$

Through N_t transmit antennas; the modulated signals $x_t^1, x_t^2, x_t^3, \dots, x_t^{N_t}$ are transmitted simultaneously.

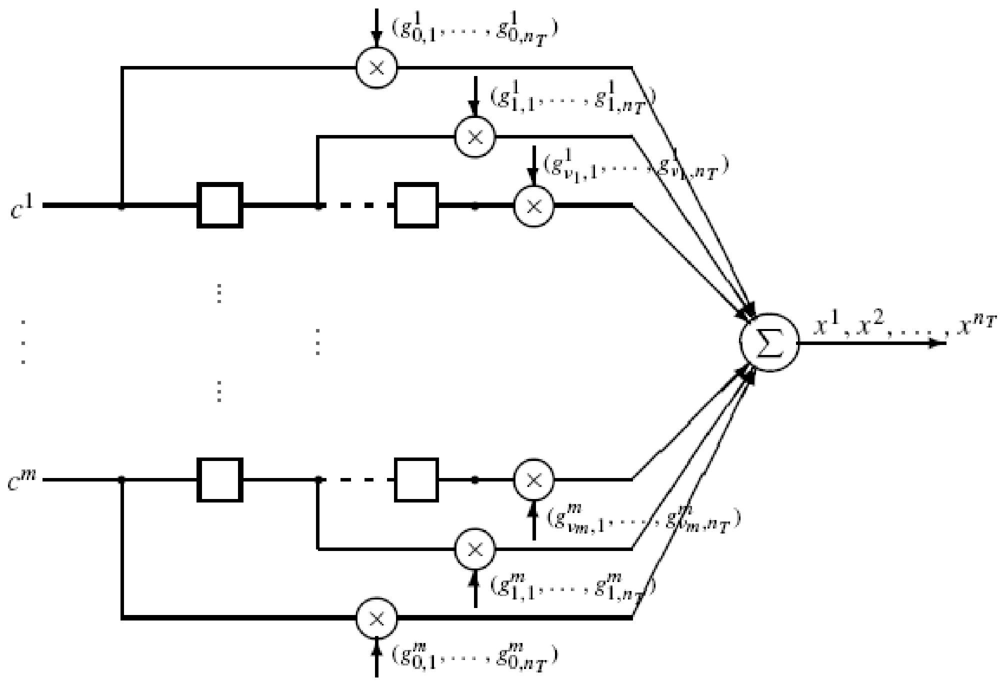

Figure 2.4: STTC Encoder **[16]**

2.4.1 Generator Description

In encoder structure as given in Figure 2.4, assume *m* is a binary input information sequences $c^1, c^2, \dots c^m$ are fed branches the encoder, it has *m* shift registers. The k-th is the input data sequence $c^k = \left(c_0^k, c_1^k, c_2^k, \ldots, c_t^k, \ldots\right), k = 1,2, \ldots m$ this data sequence passed to $k - th$ shift register than multiplied by an encoder branches coefficient set, and then the multiplier outputs from all shift registers add modulo M the encoder output is equal $x = (x^1, x^2, x^3, \ldots, x^{N_t})$. The modulo M adder can explain by the following *m* multiplication coefficient set sequences [16]:

$$\boldsymbol{g^1} = \left[\left(\boldsymbol{g_{0,1}^1}, \boldsymbol{g_{0,2}^1}, \cdots, \boldsymbol{g_{0,N_t}^1}\right), \left(\boldsymbol{g_{1,1}^1}, \boldsymbol{g_{1,2}^1}, \cdots, \boldsymbol{g_{1,N_t}^1}\right), \cdots, \left(\boldsymbol{g_{v_1,1}^1}, \boldsymbol{g_{v_1,2}^1}, \cdots, \boldsymbol{g_{v_1,N_t}^1}\right)\right] \quad 2.7$$

$$\boldsymbol{g^2} = \left[\left(\boldsymbol{g_{0,1}^2}, \boldsymbol{g_{0,2}^2}, \cdots, \boldsymbol{g_{0,N_t}^2}\right), \left(\boldsymbol{g_{1,1}^2}, \boldsymbol{g_{1,2}^2}, \cdots, \boldsymbol{g_{1,N_t}^2}\right), \cdots, \left(\boldsymbol{g_{v_2,1}^2}, \boldsymbol{g_{v_2,2}^2}, \cdots, \boldsymbol{g_{v_2,N_t}^2}\right)\right]$$

$$\vdots$$

$$\boldsymbol{g^m} = \left[\left(\boldsymbol{g_{0,1}^m}, \boldsymbol{g_{0,2}^m}, \cdots, \boldsymbol{g_{0,N_t}^m}\right), \left(\boldsymbol{g_{1,1}^m}, \boldsymbol{g_{1,2}^m}, \cdots, \boldsymbol{g_{1,N_t}^m}\right), \cdots, \left(\boldsymbol{g_{v_m,1}^m}, \boldsymbol{g_{v_m,2}^m}, \cdots, \boldsymbol{g_{v_m,N_t}^m}\right)\right]$$

where $g_{j,i}^k, k = 1, 2, 3, \ldots, m,\ j = 1, 2, 3, \ldots, v_k,\ i = 1, 2, 3, \ldots, N_t$ an element of the M-PSK signal constellation set, and v_k is the memory order of the *k*-th shift register. The encoder output at time *t* for transmit antenna *i*, denoted by x_t^i , can be computed as:

$$x_t^i = \sum_{k=1}^{m} \sum_{j=0}^{v_k} g_{j,i}^k c_{t-j}^k \qquad mod\ M, \quad i = 1, 2, 3, \ldots, N_t \qquad 2.8$$

x_t^i represent the outputs, also x_t^i are components of an M-PSK signal collection. Modulated signals shape the space-time symbol broadcast at time t.

$$x_t = \left(x_t^1, x_t^2, \ldots, x_t^{N_t}\right)^T \quad 2.9$$

The STTC M-PSK can obtain a BW efficiency of m bits/s/Hz. v is the encoder total memory, it is given as:

$$v = \sum_{k=1}^{m} v_k \quad 2.10$$

where v is the memory order for $k-th$ encoder ramification,$= 1, 2, \ldots, m,$ and value v_k for M-PSK is equal

$$v_k = \left\lfloor \frac{v+k-1}{\log_2 M} \right\rfloor \quad 2.11$$

The total number of states for the trellis encoder is 2^v . The m multiplication coefficient set sequences are also called the generator sequences.

2.5 Performance Analysis of STTC and Design Criteria on Slow Fading Channels

Different design criteria were suggested for STTCs, the rank and determinant criterion proposed by Tarokh [3], and the trace criterion which was suggested by Vucetic [17] are is the most popular techniques. These methods are briefly presented and discussed in this section.

2.5.1 Pair-wise Error Probability (PEP)

It is assume that the frame length for data transmitted is L symbols for each antenna. Space time codeword matrix is defined by dimension is $N_t \times L$, and it is represented as:

$$C = [c_1, c_2, \dots, c, \dots] = \begin{bmatrix} c_1^1 & c_2^1 & \cdots & c_L^1 \\ c_1^2 & c_2^2 & & c_L^2 \\ \vdots & & \ddots & \vdots \\ c_1^{N_t} & c_2^{N_t} & \cdots & c_L^{N_t} \end{bmatrix}$$

The PEP is defined as the probability that a decoder erroneously selects a sequence $e_t = \left(e_1^1, e_1^2, \dots, e_1^{N_t} \quad, e_2^1, \dots, e_2^{N_t} \quad e_L^1, \dots, e_L^{N_t}\right)$ as its estimate, when the transmitted sequence was in fact $c^k = \left(c_0^k, c, c_2^k, \dots, c_t^k, \dots\right), k = 1,2, \dots m$. Considering the received signal of equation 2.1, and ML concepts, this can happen if and only if,

$$\sum_{t=1}^{L} \sum_{i=1}^{N_t} \left| r_t^i - \sum_{j=1}^{N_t} h_{j,i}^t c_t^j \right|^2 \geq \sum_{t=1}^{L} \sum_{i=1}^{N_r} \left| r_t^i - \sum_{j=1}^{N_r} h_{j,i}^t e_t^j \right|^2 \qquad 2.12$$

This disparity is equivalent to

$$\sum_{t=1}^{L} \sum_{i=1}^{N_r} 2Re \left\{ \left(n_t^i\right) * \sum_{j=1}^{N_t} h_{j,i}^t \left(e_t^j - c_t^j\right) \right\} \geq \sum_{t=1}^{L} \sum_{i=1}^{N_r} \left| \sum_{j=1}^{N_t} h_{j,i}^t \left(e_t^j - c_t^j\right) \right|^2 \qquad 2.13$$

where Re {.} is the real part of a complex number.

Assume, at the receiver the perfect CSI is available, and achieve the fading channel matrix *H*, the right hand side of the equation 2.13 become into a constant that represents a modified euclidean distance between the two space-time code word matrices *c* and *e*. Denoting this constant by $d_h^2(c, e)$, the conditional pairwise error probability PEP can be represented by [5].

$$P(c,e|H) = Q\left(\sqrt{\frac{E_s}{2N_0} d_h^2(c,e)}\right) \tag{2.14}$$

where $N_0/2$ is the noise variance per dimension ,$d_h^2(c,e)$ is Euclidean distance between the two ST codeword matrices c, and e and $Q(c)$ is the Gaussian Q-function, it is defined by

$$Q(c) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-t^2/2}\, dt \tag{2.15}$$

By using the inequality

$$Q(c) \leq \frac{1}{2}\, e^{-c^2/2}\, dt \tag{2.16}$$

The conditional PEP represented by equation 2.14; it can be upper bounded as:

$$P(c,e|H) \leq \frac{1}{2} \exp\left(-d_h^2(c,e) \frac{E_s}{4N_0}\right) \tag{2.17}$$

2.5.2 Rank and Determinant Criterion

The rank and determinant criterion was the first criterion used for designing STCs. This criterion depends on the rank and determinant of the codeword difference matrix $D(c,e)$. This is defined as:

$$D(c,e) = \begin{bmatrix} e_1^1 - c_1^1 & e_2^1 - c_2^1 & \cdots & e_1^{n_t} - c_1^{n_t} \\ e_1^2 - c_1^2 & e_2^2 - c_2^2 & & e_2^{n_t} - c_2^{n_t} \\ \vdots & & \ddots & \vdots \\ \vdots & & \ddots & \vdots \\ e_1^{n_t} - c_1^{n_t} & e_2^{n_t} - c_2^{n_t} & \cdots & e_L^{n_t} - c_L^{n_t} \end{bmatrix}$$

Where c is the correct path, e is the erroneous path, and the size of matrix is $(N_t \times L)$. The distance matrix is defined as

$$A(c,e) = D(c,e) \times D(c,e)^H \qquad 2.18$$

Where H denotes the Hermitian transpose of a matrix.

rN_r is the number of independent sub channels, assume it is small, then for high SNR, the upper bound on the PEP can be found by equation 2.17 [18]. The PEP had been developed on quasi-static flat fading channels by using Chernoff bound in [7], it is as shown in the equation below:

$$P(c,e) \le \left(\prod_{i=1}^{r} \lambda_i\right)^{-N_r} \left(\frac{E_s}{4N_0}\right)^{-rN_r} \qquad 2.19$$

Where r is the rank of matrix $A(c,e)$, and λ_i are non-zero eigenvalues of $A(c,e)$.

In [18] the authors used the union bound technique as an upper bound on the code frame error probability which is computed by summing the contributions of the PEPs over all error events. When the SNR increases, the PEP reduces exponentially. So, the frame error probability is predominated by the PEP with the minimum product rN_r over all possible codeword pairs when the SNRs are high. The minimum rank should be maximized to achieve a good performance. Minimum diversity is the product of the minimum rank and the number of receive antennas rN_r.. The possible maximum value of rN_r is N_tN_r, but this may not be achieved due to the constraints on the code structure [1]. In order to minimize the error probability, we need to maximize $\prod_{i=1}^{r}\lambda_i$.

The dependence of diversity gain on PEP was an important enhancement over the basic rank and determinant criteria. In [19], Yuan proposed that the PEP is actually diversity gain dependent. For $rN_r < 4$ uses in equation 2.19 and if $rN_r > 4$ the PEP is given by:

$$P(c,e) \leq \frac{1}{4} exp\left(-N_r \frac{E_s}{4N_0} \sum_{i=1}^{r} \lambda_i\right) \qquad 2.20$$

In [20], the value 4 was considered to be the diversity gain boundary condition on PEP. This value is selected to make sure that the fading channel converges to a Gaussian channel.

The rank, determinant, and trace criteria had been developed to minimize the worst PEP. These criteria reveal that good code construction requires two properties: full diversity gains and best code gain, and have been extensively used to describe the performance of space-time codes. Although the rank criterion will lead to best diversity gain, there are no guidelines for the code gain design. This is because there are no dominant error events in quasi-static channel [21].

An alternative approach to performance analysis of space-time codes is to use the union bound technique, which allows us to analyze the contribution of all error events to the performance. Several performance analysis schemes are based on the union bound have been developed and applied in [21]. Generally, the union bound on FER is calculated from the PEP via a sum of PEP, given by:

$$P_{FER} = \sum_{c \in D} \sum_{\substack{e \in D \\ e \neq D}} P(c) P_{PEP}(c \to e) \tag{2.21}$$

Where *D* is the set of the transmitted codewords and *P(c)* is the probability of the transmitted codeword *c*.

We study the case of the small diversity gain. considering the rank is the dominant factor in determining the performances of space-time codes, to simplify the analysis, we can assume in this thesis that the codes under consideration always have a full rank; the product of nonzero eigenvalues of A is equal to the determinant of A. This leads to the following simplification of (2.19).

$$P_{PEP}(d) \leq d^{-N_r} \left(\frac{E_s}{4N_0}\right)^{-N_r N_t} \tag{2.22}$$

Where $P_{PEP}(d)$ is the PEP of the codeword error pair (*c*, *e*) with determinant *d*. Considering the $N(d)$ of determinant, through some mathematical operation of set collecting, equation (2.22) can be reduced to the weighted sum of PEP

$$P_{FER} = \sum_{d} N(d) P_{PEP}\,(d) \tag{2.23}$$

Where *N(d)* is the multiplicity of error paths for small diversity gain, which is defined as the average number of error events with determinant *d*.

Thus, for the small diversity gain, the FER, given by substituting (2.22) into (2.23), is

$$P_{FER} \leq \sum_{d} N(d) d^{-N_r} \left(\frac{E_s}{4N_0}\right)^{-N_t N_r} \tag{2.24}$$

From the equation 2.24 above, it can been seen that the FER differs linearly with *N(d)*. Also, the FER decreases, when, on the other side *d* increases. Then, when the *N(d)* increases, the FER also increases. The coding loss caused by the determinant spectrum (η) is defined as:

$$\eta = \sum_{d} N(d) d^{-N_r} \qquad 2.25$$

Based on (2.24) and (2.25), code design criteria for the small diversity gain are set to yield a minimum FER as follows:

- Rank criteria: in order to achieve the maximum diversity $N_t N_r$, the matrix A has to be full rank for any codewords *c* and *e*.
- To achieve η optimal coding loss caused by the determinant spectrum, the code that could provide the minimum value of ηshould be chosen.

2.5.2.1 Rank Criteria

The diversity gain of an STTC is optimized by the rank criterion. Assume $A_{(c,e)}$ has full rank over the set of distinct codewords so that maximum diversity of is achieved. To illustrate this criterion [57], consider a 4-PSK system where the transmitted codeword is *c*=210013, and the erroneous codeword at the receiver decided upon is *e* = 130102. Figure 2.5 gives the 4-PSK signal constellation. In this example, n_t= 2 and the message length is L = 3. The 2 x 3 difference matrix is

$$D_{(c,e)} = \begin{bmatrix} -1-j & 1-1 & j-1 \\ j+j & 1-j & -j+1 \end{bmatrix}$$

$$D_{(c,e)} = \begin{bmatrix} -1-j & 0 & j-1 \\ 2j & 1-j & -j+1 \end{bmatrix}$$

The rank of $D_{(c,e)}$ is equal to 2 and it is same value of rank for $A_{(c,e)}$. For this system with N_t= 2 and N_r= 1 the diversity gain =2.

2.5.2.2 Determinant Criteria

The determinant criterion optimizes the coding gain. Recall that r is the rank of $A_{(c,e)}$ .The Coding gain corresponds to the minimum r th roots of the sum of the determinants of all $r \times r$ principal cofactors of $A_{(c,e)} = D_{(c,e)} D_{(c,e)}{}^{H}$ taken over all pairs of distinct codewords c and e [1]. Now $(\lambda_1 \lambda_2 \cdots \lambda_r)$ is the absolute value of the sum of the determinants of all principal $r \times r$ cofactors of $A_{(c,e)}$. Thus if a diversity advantage of $N_r r$ is achieved, the coding gain is $(\lambda_1 \lambda_2 \cdots \lambda_r)^{1/r}$. So if maximum diversity of $N_r N_t$ is the design target then we have to maximize the minimum determinant of $A_{(c,e)}$.

2.5.3 Trace Criterion

The trace criterion was introduced in [19], where the authors proposed a large number of independent subchannels $rN_r \geq 4$. Here, at high SNR, the upper bound on the PEP was expressed as:

$$P_{FER} \leq \frac{1}{4} \exp\left(-N_r \frac{E_s}{4N_0} \sum_{i=1}^{r} \lambda_i\right) \qquad 2.26$$

In order to achieve minimum PEP, the minimum sum of all eigenvalues of matrix $A_{(c,e)}$should be maximized. The trace of the matrix can be expressed as:

$$tr\big(A(c,e)\big) = \sum_{i=1}^{r} \lambda_i = \sum_{i=1}^{N_t} A^{i,i} \qquad 2.27$$

Where $A^{i,i}$ the elements of diagonal for matrix are $A(c,e)$, $A^{i,i}$ is determind by the equation:

$$A^{i,i} = \sum_{t=1}^{L} (c_t^i - e_t^i)(c_t^i - e_t^i)^* \qquad 2.28$$

Where $(.)^*$ represents the Hermitian. On the basis of equations 2.32 and 2.33, the trace of the matrix $A(c,e)$can be expressed as:

$$tr\big(A(c,e)\big) = \sum_{i=1}^{N_r} \sum_{t=1}^{L} \left|c_t^i - e_t^i\right|^2 \qquad 2.29$$

We see that the trace of matrix $A(c,e)$ is equal to the squared minimum Euclidean distance between codewords c and e.

The rank, determinant, and trace criteria had been used to reduce the worst PEP. These criteria indicate that good codes should possess full diversity gains and maximum code gain. For code gain design we cannot find guidelines to design it, because there are no dominant error events in quasi-static channel [22].

2.5.4 Symmetry Properties of STTCs.

Recently in [23], the concept of quasi-regularity was extended for STTCs over slow fading channels. It was stated that, for quasi-regular codes, the determinant spectrum can be computed assuming that the all-zero codeword was transmitted. This symmetry property facilitates the evaluation of the determinant spectrum.

A trellis coded modulation scheme is said to be quasi-regular if

1. It is encoder is liner.
2. The distance polynomial corresponding to any error vector is independent of the state.

The distance polynomial $P_{s,s',e}(x)$ is defined as:

$$P_{s,s',e}(x) = \sum_{c/s} p(c/s) x^{d[y(c),y(c\oplus e)]} \qquad 2.30$$

Where $p(c/s)$ is the probability of the signal vector c given that the encoder is in state s and $d[y(c), y(c\oplus e)]$ is the Euclidean distance between the signal y(c) in s corresponding to codeword c and the signal $y(c\oplus e)$ in s corresponding to codeword $(\boldsymbol{c}\oplus\boldsymbol{e})$.

2.6 Code Constructions

There are many ways which could be used to represent the STTCs, such as the trellis form or generator matrix form. Trellis form presents most codes, but the generator matrix form is the best for a systematic code search [13].

2.6.1 Code Construction of 4-State 4-PSK STTC

Figure 2.5 below shows a signal constellation diagram for 4-PSK [24]. The signal phase contains PSK information. One of four equally spaced values, like $0, 2\pi/4, 4\pi/4$ and $6\pi/4$ is taken by phase. A initial code typically represents these values. Also, signal points are described as 0, 1, 2, and 3. Figure 2.7 below shows the encoder structure of a 4-State 4-PSK STTC with input bits to the upper and lower branches. Also, v_1 and v_2 are the memory arrangement of the upper and lower branches respectively. These are essentially shift registers. The shifts register in the encoder, stores the previous transmitted bits. The memory order of the encoder is represented by the length of the shift register [13].

Example 1:

The generator matrix for TSC code of a 4-state space-time trellis coded QPSK scheme with 2 transmits antennas is:

$$\mathbf{g}_1 = [(02), (20)]$$
$$\mathbf{g}_2 = [(01), (10)]$$

The trellis structure of the code is as shown in Figure 2.6. The trellis consists of $2^v =$ 4 states, represented by state nodes. The encoder takes m = 2 bits as its input at each time. There are $2^m = 4$ branches leaving from each state corresponding to four different input patterns, $c_t^1 c_t^2 / x_t^1 x_t^2$ labels each branch, where a pair of encoder input bits are $c_t^1$and c_t^2, and the two coded QPSK symbols which are transmitted through antennas 1and 2 are $x_t^1$and x_t^2represent respectively. The row listed next to a state node in Figure 2.6 indicates the branch labels for transitions from that state corresponding to the encoder inputs 00, 01, 10, and 11, respectively.

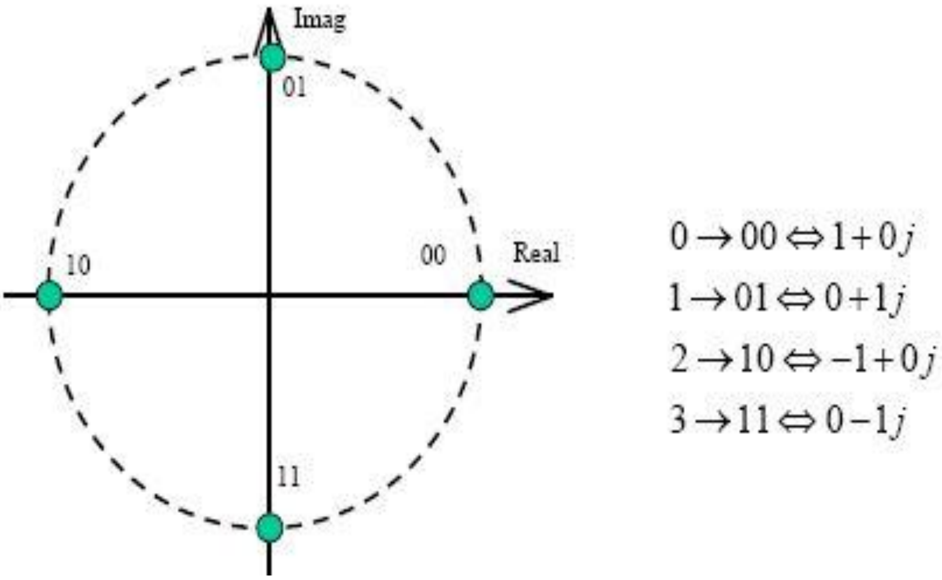

Figure 2.5: Signal Constellations for 4-PSK [13]

The 4-State rate of 2 b/s/Hz 4-PSK scheme has trellis structure as shown in Figure 2.6:

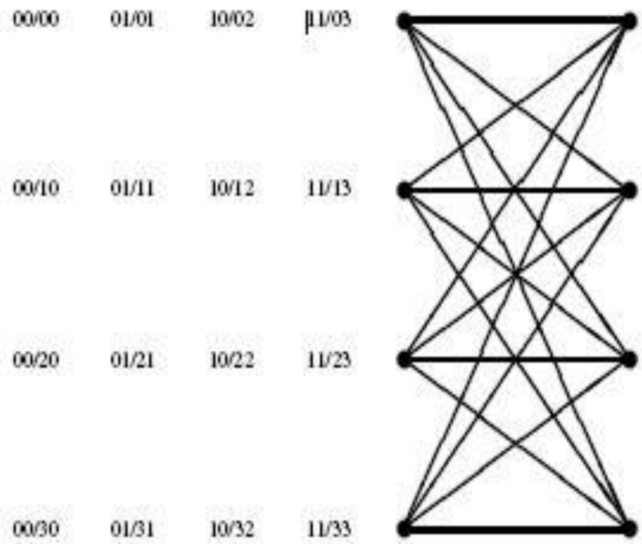

Figure 2.6: Trellis Diagram 4-state, 4-PSK [5]

The encoder designs for the 4-PSK scheme with $N_t = 2$ and $N_r = 1$ is shown in Figure 2.7 the two binary information inputs c_t^1 and c_t^2 at t time, are fed into the encoder branches the memory order of the encoder branches (upper and lower branches) are v_1 and $v_2$respectively, where $v = v_1 + v_2$ and the number of state is 2^v and v_i was explained before in section 2.4.

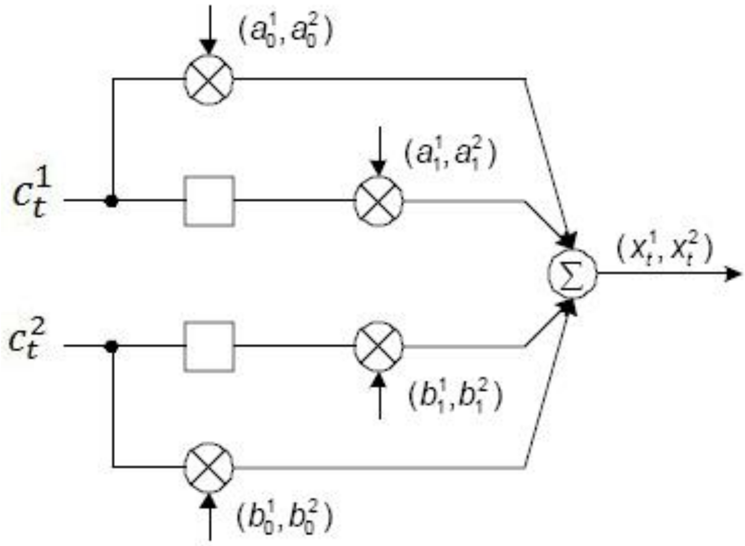

Figure 2.7: Encoder Structure for 4-state 4-PSK [12]

The trellis diagrams for TSC 8-state and 16-state 4-PSK are as in Figure 2.8 and Figure 2.9. In 8-state 4PSK, the encoder contents three shift register therefor the number of state $2^v = 8$ state and the generator matrix as:

$$g_1 = [(0\ 2)(2\ 0)]$$
$$g_2 = [(0\ 1)(1\ 0)(2\ 2)]$$

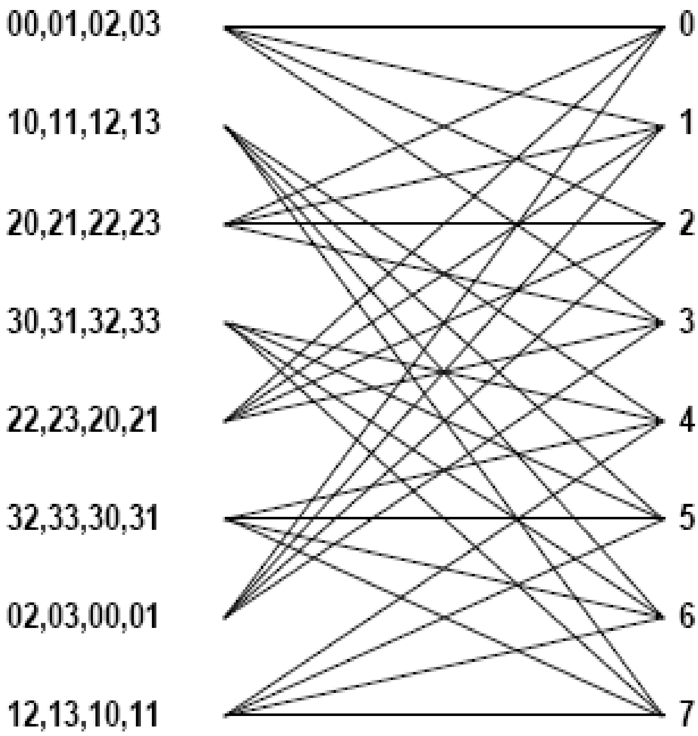

Figure 2.8: Trellis Diagram for 8-state 4-PSK [3]

In 8-state 4PSK, the encoder contents four shift register therefor the number of state $2^v = 16$ state and the generator matrix as:

$$g_1 = [(0\ 2)(2\ 0)(0\ 2)]$$
$$g_2 = [(0\ 1)(1\ 2)(2\ 0)]$$

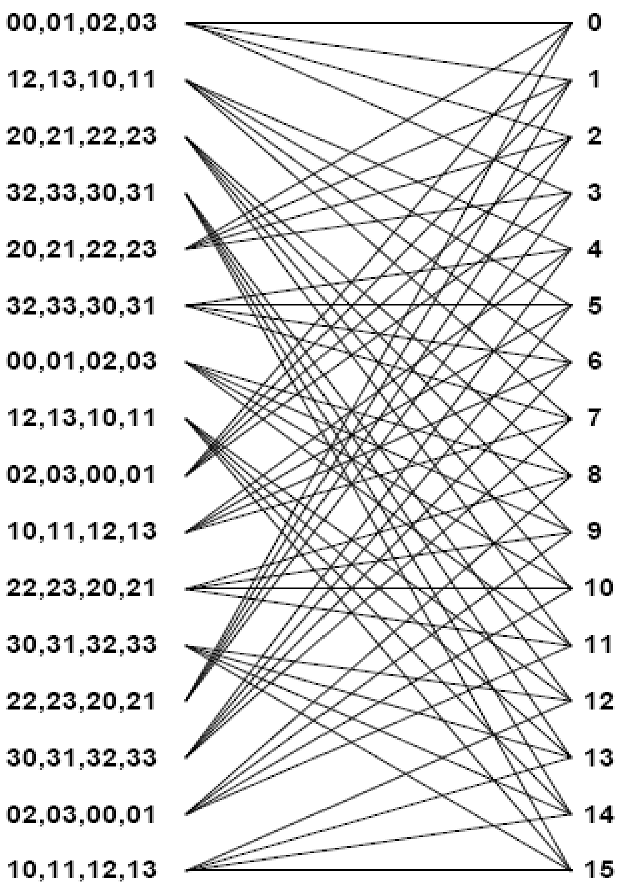

Figure 2.9: Trellis Diagram for 16-state 4-PSK [3]

2.6.2 Code Construction of 8-State 8-PSK STTC

Figure 2.8 below shows signal constellation for 8-PSK. The trellis diagram and the encoder for the 8-PSK 8-state trellis code are given in Figures 2.9 and 2.10. The encoder for 8-PSK is same as that of 4-PSK except the input in 8-psk is $N_t = 3$, that means three groups of coefficients. The third input corresponds to a branch of memory v_3 .

Example 2**:**

Assume that the generator matrix for 8-PSK, 8 state with two transmit antenna is

g_1 = [0 4 ; 4 0]
g_2 = [0 2 ; 2 0]
g_3 = [0 1 ; 5 0]

The number of shift register equal $2^v = 8$ then, v=3. That means we have three branches in encoder which contains a shift register as shown in Figure bellow.

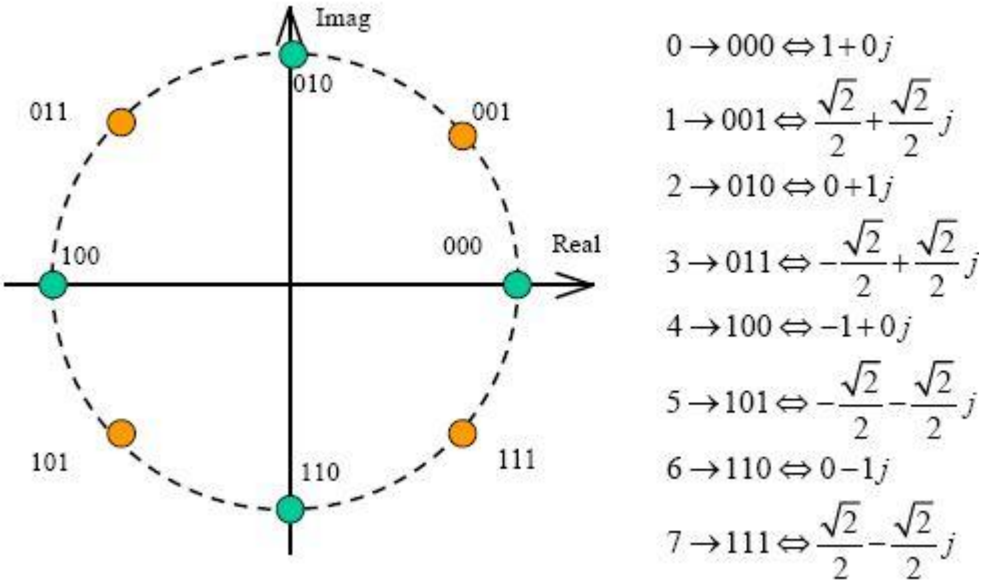

Figure 2.10: 8-PSK Signal Constellations **[13]**

The encoder design for the 8-PSK scheme with $N_t = 2$ and one $N_r = 1$ is shown in Figure 2.12. The three binary information inputs arec_t^1, c_t^2 and c_t^3 at t time, which are fed the encoder branches. The memory order of the encoder branches (upper and lower branches) arev_1, v_2 and $v_3$respectively,

$$v = v_1 + v_2 + v_3. \quad 2.31$$

Where v is the total of memory branches.

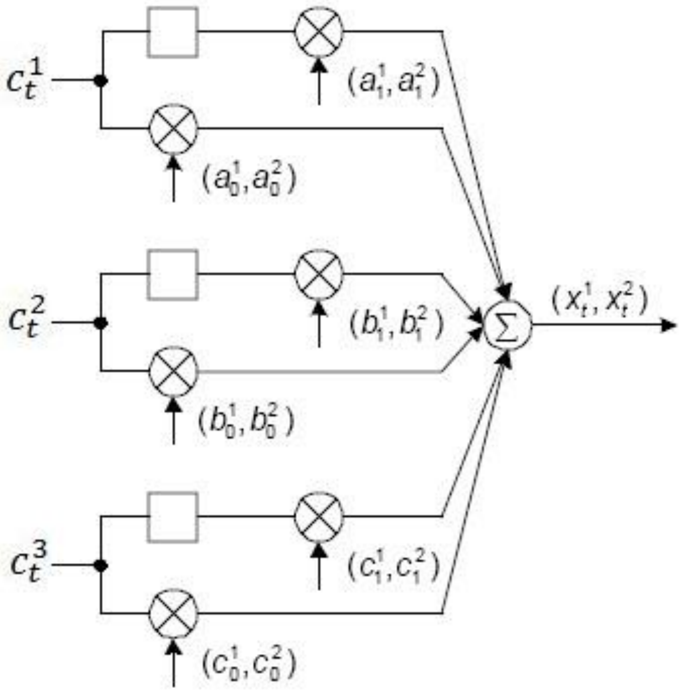

Figure 2.11: Encoder Stretcher for 8-PSK [12]

Trellis diagram for 8-PSK 8-State in this example is:

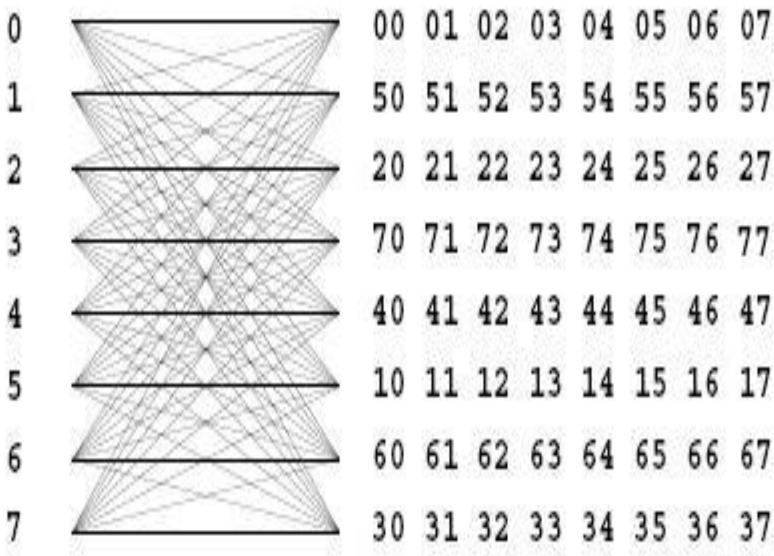

Figure 2.12: Trellis Diagram for TSC Code 8-state 8-PSK [3]

2.7 STTCs decoder

STTC Decoder utilizes the Viterbi algorithm in order to perform maximum likelihood decoding (ML). It is presumed that perfect CSI is available at the receiver. A branch is labeled by the symbol x_t, the branch metrical is calculated as the squared Euclidean distance among the assumption received symbols and the real received signals as:

$$\sum_{j=1}^{n_T} \left| r_t^j - \sum_{i=1}^{n_T} h_{j,i}^t x_t^i \right|^2 \qquad 2.32$$

The Viterbi algorithm chooses the path with the minimum path as the decoded sequence [25].

In the next chapter we will study the effect of the determinant criteria on the FER and besides we will see the performance analysis and simulation result for our work.

Chapter 3

PERFORMANCE ANALYSIS AND SIMULATION RESULTS

When first invented, STTCs for small number of independent subchannels were constructed so that, the rank and the minimum determinant of the distance matrix are maximized over a slow fading channel. Later in 2005, an improved design criteria [7] that leads to the construction of better STTCs taking into account the first few lines of the determinant spectrum for 4-PSK was presented. In this chapter, the performance assessment of some known 4 and 8-state 4-PSK as well as 8-state 8-PSK STTC schemes are presented. Some new 4PSK and 8PSK STTC schemes with better performance than the known ones are constructed. The performance of all schemes is measured using simulation; in addition, for some schemes that possess the quasi-regularity property, the first 5 lines of the determinant spectrum are presented and compared.

The simulation program which is implemented in MATLAB uses 130 symbols per frame in over quasi-static flat fading channel with two transmit and one receive antennas. An ML Viterbi decoder with ideal channel state information is used at the receiver.

3.1 Performance Analysis and Simulation Results for 4-PSK schemes with 4 and 8-state

First, the performance of the known 4-state 4PSK TSC and new code codes is evaluated twice; first the frame error rate (FER) is obtained using simulation and the result are as plotted in Figure 3.2, second, the first five lines of the determinant spectrum are presented and tabulated as in Table 3.1

The generators matrix of the new code is found as:

$$g_1 = [(0\ 2)(2\ 0)]$$
$$g_2 = [(0\ 1)(2\ 0)]$$

It is and trellis diagram is as sketched in Figure 3.1

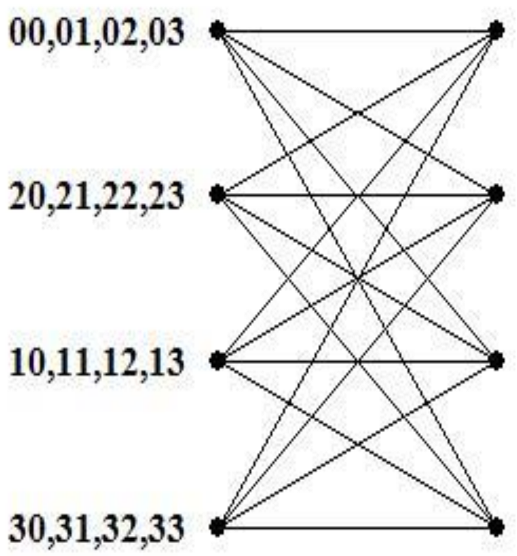

Figure 3.1: Trellis Diagram for the New Code 4-state 4-PSK

It was checked that the TSC and new 4-state 4PSK codes are quasi-regular while the LP code is not.

As seen form Table 3.1 the second and fifth spectral lines of the TSC code are more dominant as they have large multiplicities than the other, whereas for the new code scheme, all spectral lines have comparable contributions.

Table 3.1: First five lines of the determinant spectrum for 4-state 4-PSK STTC with ($N_t = 2$ and $N_r = 1$)

Codes	Generator matrices	R	Det	η	d_1 N_1	d_2 N_2	d_3 N_3	d_4 N_4	d_5 N_5
TSC	$G_1 = [(0\ 2)\ (2\ 0)]$ $G_2 = [(0\ 1)(1\ 0)]$	2	4	1.282	4 2	12 4	16 1	20 2	28 8
New code	$G_1 = [(0\ 2)(2\ 0)]$ $G_2 = [(\ 0\ 1)(2\ 0)]$	2	4	0.908	4 1	8 2	12 1	16 2	20 4

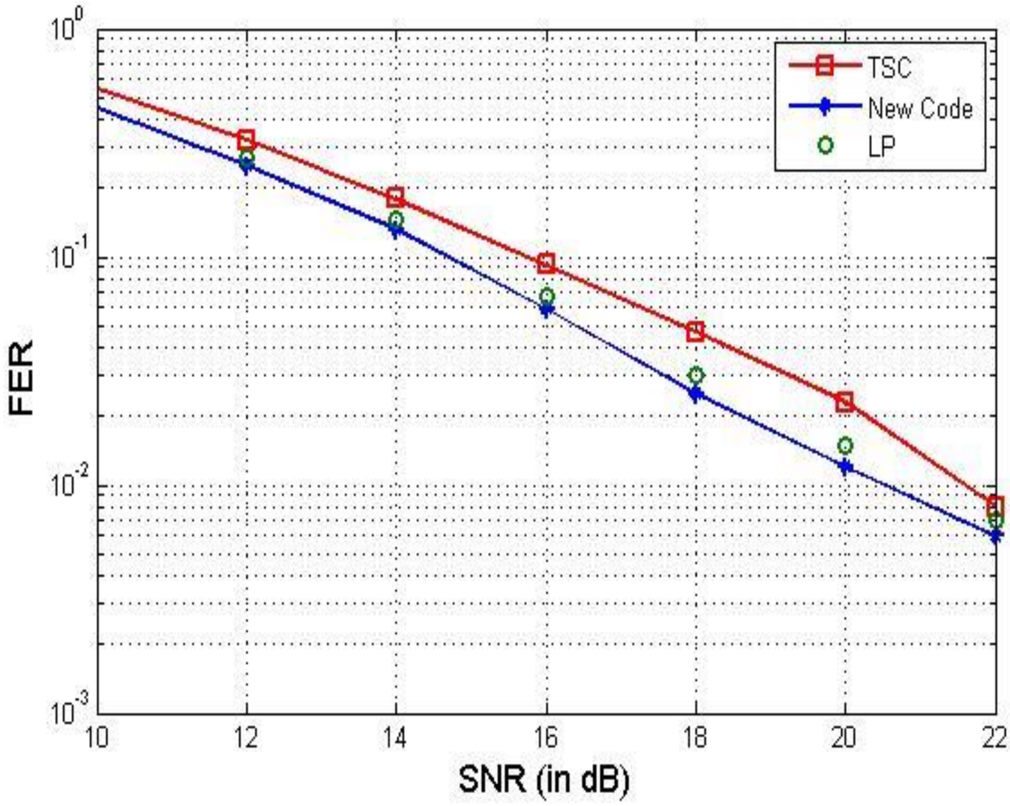

Figure 3.2: The Performance of STTC for 4-State 4-PSK with $N_t = 2$ and $N_r = 1$.

From the Figure 3.2 above, we see the new code has better performance than other codes, and the FER decreases when the SNR increases. These results are compatible with the results of Table 3.1 in η and multiplicity$N(d)$. From the table also, we see η for the new code is less than that TSC code, that means when η is small the code performance becomes better.

The simulation results in Figure 3.2 indicted that the new 4-state 4PSK code outperforms the corresponding TSC and LP codes by 0.3 dB and 1 dB respectively at a FER of 10^{-2}.

Similar simulation results were obtained for the TSC and LP 8-state 4PSK schemes together with those of the corresponding new code, whose trellis diagram is shown in Figure 3.3. The results are as shown in Figure 3.4.

The first five lines of the DS are also computed for the 8-state 4PSK TSC schemes as tabulated in Table 3.2. The results obtained indicate that for the TSC code, the forth spectral lines is more dominant than the others

The simulation result in Figure 3.4 indicted that the new 8-state 4PSK code outperforms the corresponding TSC and LP codes by 0.8 dB and 2.5 dB respectively at a FER of 10^{-2}.

The generator matrix of the new code is found as:

$$g_1 = [(0\ 2)(2\ 0)]$$
$$g_2 = [(2\ 3)(3\ 2)(2\ 2)]$$

and the trellis diagram is shown in Figure 3.3:

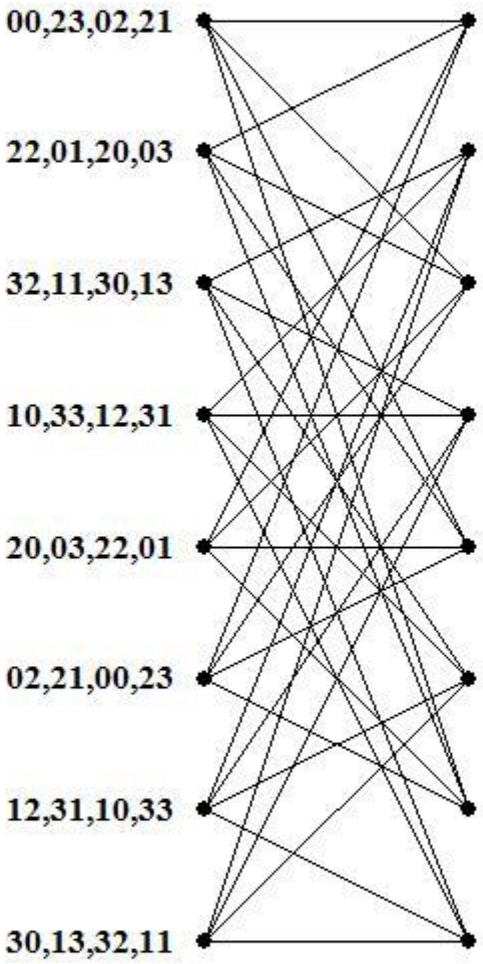

Figure 3.3: Trellis Diagram for New Code of the 8-state 4-PSK

Table 3.2: First five lines of the determinant spectrum for TSC 8-state 4-PSK STTC with ($N_t = 2$ and $N_r = 1$)

Code	Generator matrices	r	Det	η	d_1 N_1	d_2 N_2	d_3 N_3	d_4 N_4	d_5 N_5
TSC	$G_1 = [(0\ 2)\ (2\ 0)]$ $G_2 = [(0\ 1)(1\ 0)(2\ 2)]$	2	12	0.806	12 2	16 1	20 4	28 9	36 2

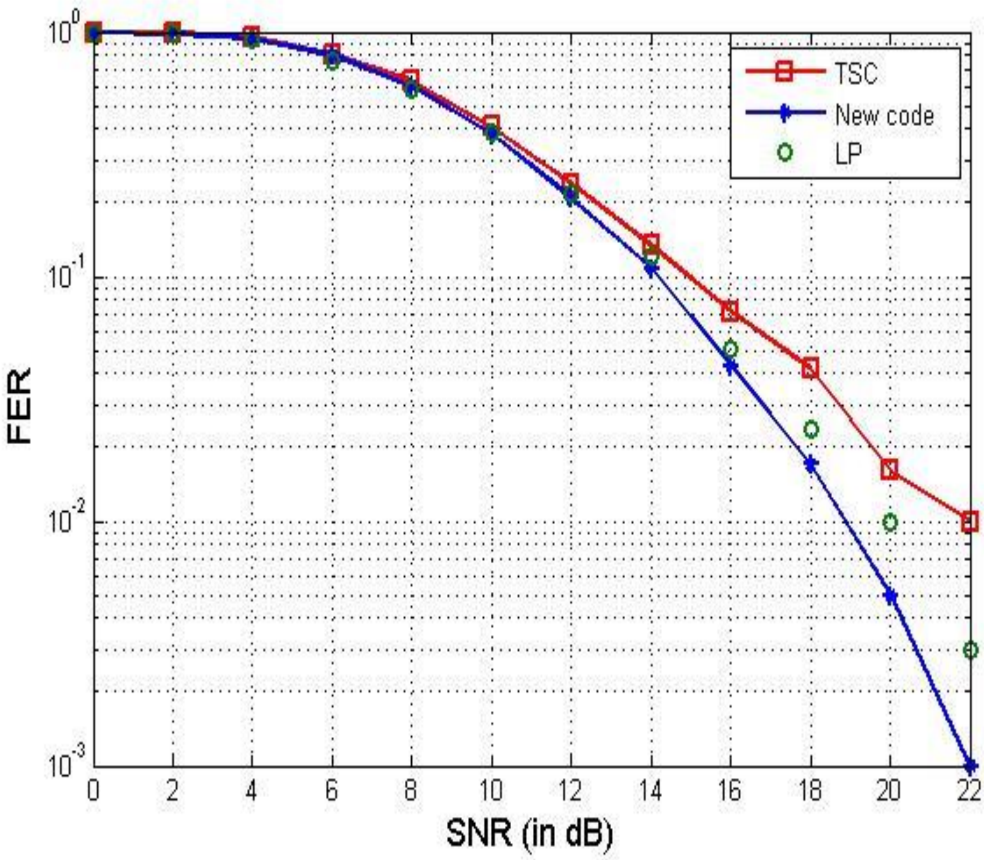

Figure 3.4: Performance of STTC for 4-state 4-PSK with $N_t = 2$ and $N_r = 1$

3.1 Performance Analysis and Simulation Results of 8-PSK

The performance of the known 8-state 8PSK TSC code is evaluated using simulation and compared to the new scheme whose generator matrix is given as

$$g_1 = [5 \;\; 0\,;\; 3 \;\; 5]$$
$$g_2 = [2 \;\; 4\,;\; 4 \;\; 2]$$
$$g_3 = [2 \;\; 3\,;\; 3 \;\; 2]$$

and corresponding trellis diagram is as shown in Figure 3.5. The results obtained indicate that the new code outperforms the TSC code by 2 dB at a FER of 10^{-2}.

It was stated that [23] the 8-state 8PSK TSC code is quasi regular and the DS can be computed assuming that the all zero path was transmitted. The first five lines of the DS for the TSC code are presented and tabulated as shown in Table 3.3. The new code was checked to be nonquasi-regular.

The trellis diagram for the 8-state 8PSK new code is shown in Figure 3.5.

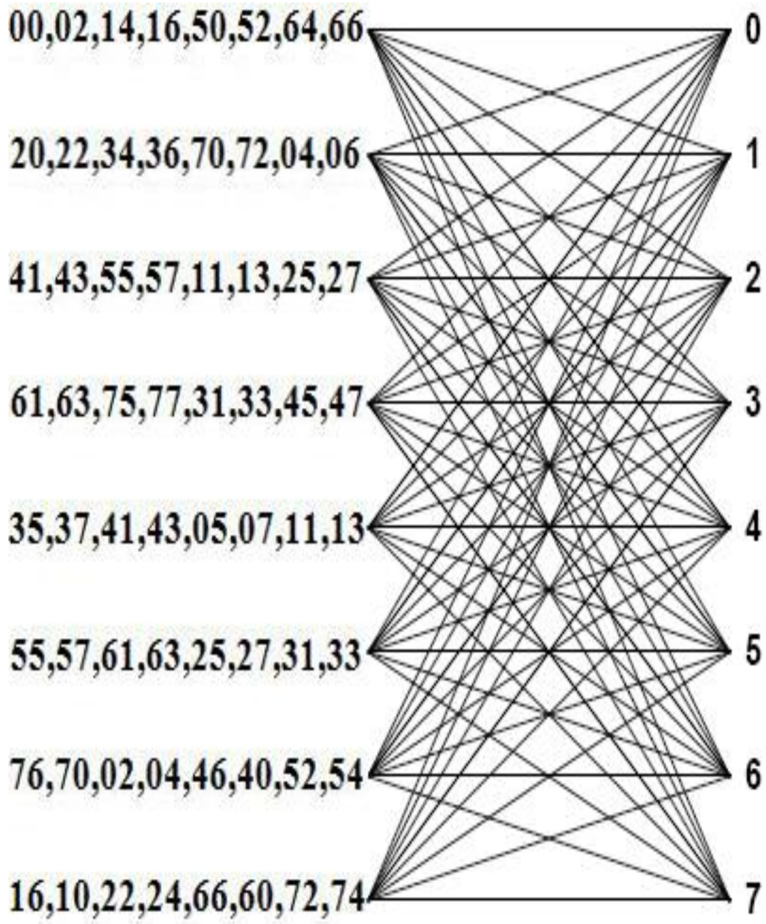

Figure 3.5: Trellis Code for New Code of the 8-state 8-PSK

Note that the 8-PSK signal constellation has the signal selectors 011 and 111 which are associated with more than one Euclidean distance Figure 3.6 and this should be taken into account while determining the determinant spectrum. Considering the complex plane with horizontal real, and vertical imaginary axes, the Euclidean distances associated with 011 are $d_1 = 1 - \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$ and $d_2 = 1 + \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$ while those associated with 111 are $d_1 = 1 - \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}$ and $d_2 = 1 + \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$ [26].

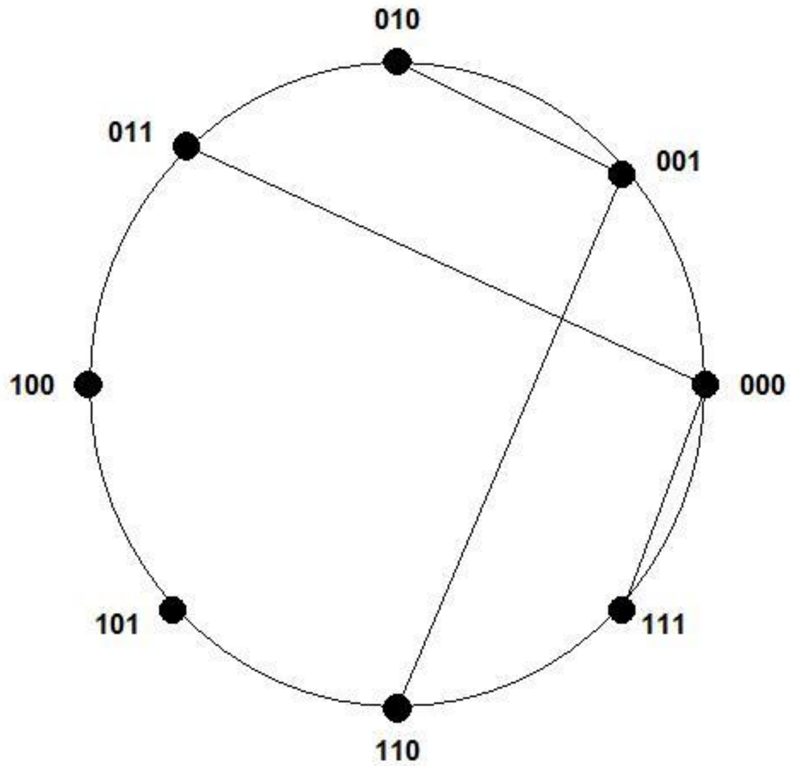

Figure 3.6: Different Distances Corresponding to Signals 3(011) and 7(111) [23]

To demonstrate, consider the following first error event.

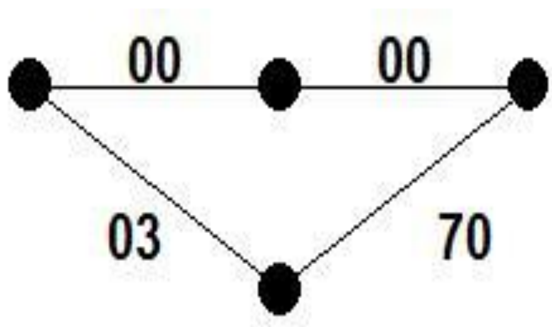

Figure 3.7: Sample Error Event with Length L=2 [23].

Taking into account the Euclidean distances corresponding to signal selector's o11 and 111, this error event yields three different determinant values with associated frequency of occurrence. The three obtained determinants are 0.343, 2 and 11.66 and the multiplicity 0.25, 0.5 and 0.25 receptivity.

Table 3.3: First five lines of the determinant spectrum for 8-state 8-PSK STTCs with ($N_t = 2$ and $N_r = 1$)

Line	Generator Matrices 8-state 8-PSK of TSC Code
	$G_1 = [0\ \ 4\ ;\ 4\ \ 0]$ $G_2 = [0\ \ 2\ ;\ 2\ \ 0]$ $G_3 = [0\ \ 1\ ;\ 5\ \ 0]$
(d_1, N_1)	(0.343,0.5)
(d_2, N_2)	(1.03,0.25)
(d_3, N_3)	(1.715,0.0312)
(d_4, N_4)	(2,3)
(d_5, N_5)	(2.686,1.437)

It can be note that the first and fourth spectral line of the DS is the most dominant for the 8-state 8PSK TSC code.

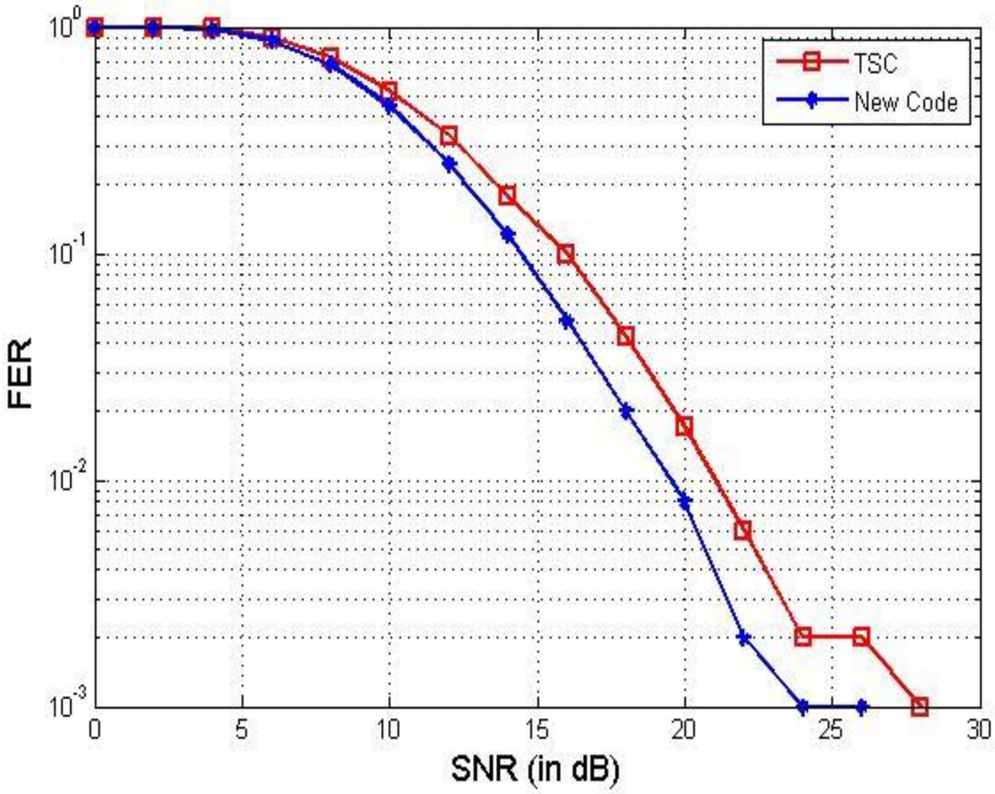

Figure 3.8: Comparison between our New Code and TSC code for 8-state 8-PSK with $N_t = 2$ and $N_r = 1$.

From Figure 3.8 above, new code performs batter than the TSC code especially when the SNR increases. The new 8-state 8PSK code outperforms the TSC code by 2dB at a FER of 10^{-2}.

Chapter 4

CONCLUSION

In this thesis, the 4-state and 8-state 4 PSK and 8PSK codes constructed for the slow fading channel were studied. It was stated that when the number of independent subchannels is small ($rN_r < 4$), the performance of a space time trellis code is measured from it is determinant spectrum. For trellis coded modulation schemes over the AWGN channel, the first few lines of the distance spectrum were sufficient to assess their performance, however, for STTC on slow fading channels, there is no dominant error event and many lines of the determinant spectrum should be taken into account especially for small determinant. This necessitates the existence of efficient algorithms for determinant spectrum computation.

It is also observed that for some attempts to improve the code performance, the quasi-regularity of some codes was lost.

The improved design criteria based on FER for space-time codes have been applied. The new 4-state 4PSK code has been designed and simulated. The results show that the new code outperforms the TSC code and still preserves the quasi-regularity property.

The 8-state 4PSK and 8PSK schemes have shown superior performance to that of the existing TSC and LP schemes however, symmetry was not preserved.

As a future work, 16-state 8PSK codes with better performance than the existing TSC code can be searched and a general algorithm for computing the DS for non-symmetric codes can be applied.

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APPENDICES

Appendix A: The Effect of The Determinant Spectrum on The Performance

Assume that the determinant spectrum up to the first five lines has 100% effect on the performance. The formula below is used to find the effect of each spectral.

$$d_n = \frac{\frac{N(d_n)}{d_n}}{\eta} \times 100\%$$

Table A.1: The Effect of First Five Lines of The Determinant Spectrum for TSC and New Code for 4-state 4 PSK

Det	η	d_1 N_1	d_2 N_2	d_3 N_3	d_4 N_4	d_5 N_5
TSC code 4-state 4PSK	1.282	4,2	12,4	16,1	20,2	28,8
Effect%		39%	26%	4.87%	0.078%	22.28%
New code 4-state 4PSK	0.908	4,1	8,2	12,1	16,2	20,4
Effect%		27.5%	27.5%	9.17%	13.76%	22.%

Table A.2: The Effect of First Five Lines of The Determinant Spectrum for TSC Code for 8-state 4 PSK

Det	η	d_1 N_1	d_2 N_2	d_3 N_3	d_4 N_4	d_5 N_5
TSC 8-state 4 PSK	0.8062	12,2	16,1	20,4	28,9	36,2
Effect%		20.6%	7.75%	24.8%	39.9%	6.9%

Table A.3: The Effect of First Five Lines of The Determinant Spectrum for TSC Code 8-state 8PSK

Line	TSC 8-state 8PSK	η =3.7536
(d_1, N_1)	(0.343,0.5)	38%
(d_2, N_2)	(1.03,0.25)	6.5%
(d_3, N_3)	(1.715,0.0312)	0.5%
(d_4, N_4)	(2,3)	39.96%
(d_5, N_5)	(2.686,1.437)	14.25%

Appendix B: Matlab Codes

```
M=input('Please enter the number of constellation points (e.g 4,8):')

m = log2(M); % Number of bits per symbol.

Scheme = 'M-PSK'; % Modulation scheme to use. (M-PSK or M_QAM).

Fr =1000; % Number of FRAMEs.

N_States=input('Please enter the number of states (e.g 4,8,16,32):')

SNRdB = 0:2:30; % SNR in dB.

SNR = 10.^(SNRdB./10); % SNR in linear scale.

N_Transmit=input('Please enter the number of transmit antennas (e.g 1,2):')

N_Receive =input('Please enter the number of receive antennas (e.g 1):'); % Number
of Rx Antennas.

N_Receive2=N_Receive;

Criterion = 'R&Dc'; % Criteria for GENERATORs. ('R&Dc' or 'Tc').

Code1 = 'BBH'; % Code used for GENERATORs.('TSC').

Code2 = 'TSC'; % Code used for GENERATORs.('BBH').

Code3 = 'new'; % Code used for GENERATORs.('LP').

Code4='New';   % Code used for GENERATORs.('New').


%*******************************************************

%                 FIRST START

%*******************************************************

for i = 1 : Fr

% Complex White Gaussian Noise with mean ZERO and variance ONE.

% Variance ONE means variance of 1/2 per dimension (Real & Imaginary).
```

```
WGN1 = (randn(N_Receive, length(Data_In_Sy1)) + 1i*randn(N_Receive, length(Data_In_Sy1)));
WGN1 = sqrt(1/2) .* WGN1;
% Rayleigh Fading.
% Modelled as an i.i.d complex Gaussian random variable with zero mean
% and variance 1/2 per dimension.
% Variance 1/2 per dimension means that variance of real and imaginary
% parts is 1/2 each thereby making the overall variance equal to 1, as
% ( ( sqrt(1/2)^2 ) + ( sqrt(1/2)^2 ) ) = 1.
H1 = sqrt(1/2) .* (randn(N_Receive, N_Transmit) + 1i*randn(N_Receive, N_Transmit));
% Channel State Information is at the Transmitter.
if Tx_CSI1 == 1
Hstar1 = (H1').';
W1 = zeros(N_Receive, N_Transmit);
for j = 1 : N_Receive
W1(j, :) = (Hstar1(j, :) ./ sqrt(sum(abs(H1(j,:)).^2)));
end
H1 = H1 .* W1;
else
H1 = sqrt(Es1/N_Transmit) .* H1;
end
% Faded Signal.
faded_r1 = (H1 * modulated_X1);
for j = 1 : length(SNRdB)
```

```
% Additive White Gaussian Noise with mean ZERO and variance No/2.
AWGN1 = N_sd1(j) * WGN1;
% Received Signal = Faded Signal + AWGN Noise.
r1 = faded_r1 + AWGN1;
% Demodulated & Decoded signal.
% [estimated_Symbols estimated_Bits] = Decoder(M, Tr, H, r, Scheme);
N_r1 = size(r1,1); % Number of Rx Antennas.
Nstates1 = size(Tr1, 2)/M; % Number of STATES.
Symbols2 = size(r1, 2); % Number of received SYMBOLs.
%% Decode using VITERBI ALGORITHM (MAXIMUM LIKELIHOOD DECODING).
X1 = Modulator(Tr1, M, Scheme);
hX1 = (H1 * X1);
BM1 = zeros(Symbols2 , Nstates1 * M);
for k = 1 : Symbols2
for n = 1 : N_r1
BM1(k, :) = BM1(k, :) + ( abs( r1(n, k) - ( hX1(n, :) ) ) ).^2;
end
end
[estimated_Symbols1 estimated_Bits1] = Viterbi_Decoder(r1, Tr1, M, BM1);
% Count Error Bits.
Error_Bits1 = sum(xor(Data_In1, estimated_Bits1));
% Count Error Symbols.
Error_Symbols1 = sum(bsxfun(@ne, Data_In_Sy1, estimated_Symbols1));
% Calculate BER.
```

```
BER1(j) = BER1(j) + Error_Bits1;

% Calculate SER.

SER1(j) = SER1(j) + Error_Symbols1;

% Calculate FER.

FER1(j) = FER1(j) + ( Error_Bits1 & 1 ) ;

pause(.0000000001)

end

end

BER1 = BER1 ./ (Fr * length(Data_In1));

SER1 = SER1 ./ (Fr * length(Data_In_Sy1));

FER1 = FER1 ./ Fr;

    START OF 4 STATES, 2T, 1R

% *****************************************************

%*************************************************************
*******

if M == 4

if states == 4

if N_t == 2

if strcmpi(criteria, 'R&Dc')

if strcmpi(code, 'TSC')

varargout(1) = { [0 2 ; 2 0] };

varargout(2) = { [0 1 ; 1 0] };

elseif strcmpi(code, 'BBH')

varargout(1) = { [2 2 ; 1 0] };

varargout(2) = { [0 2 ; 3 1] };
```

```
elseif strcmpi(code, 'PL')

varargout(1) = { [0 1 ; 2 0] };

varargout(2) = { [0 2 ; 1 0] };

end ******************************************************

%                    NEW

******************************************************

elseif strcmpi(code, 'New')

varargout(1) = { [2 1 ; 1 2] };

varargout(2) = { [0 2 ; 2 1] };

%******************************************************

%        END OF 4 STATES WITH 2T & 1R

% ******************************************************

******************************************************

%START          OF          8          STATES,          2T,          1R

******************************************************

******************************************************

elseif states == 8

if N_t == 2

if strcmpi(criteria, 'R&Dc')

if strcmpi(code, 'TSC')

varargout(1) = { [0 2 ; 2 0] };

varargout(2) = { [0 1 ; 1 0 ; 2 2] };

elseif strcmpi(code, 'BBH')

varargout(1) = { [2 2 ; 2 0] };

varargout(2) = { [0 1 ; 1 0; 2 2] };
```

```
elseif strcmpi(code, 'LP')

varargout(1) = { [0 2 ; 2 1] };

varargout(2) = { [2 0 ; 1 3; 0 2] };

end

%*****************************************************

*******************************************************

%                    NEW

% *******************************************************

%******************************************************

elseif strcmpi(code, 'New')

varargout(1) = { [0 2 ; 2 0] };

varargout(2) = { [2 3 ; 3 2; 2 2] };

%******************************************************

%                    END

%******************************************************

elseif M == 8

if states == 8

if N_t == 2

if strcmpi(criteria, 'R&Dc')

if strcmpi(code, 'TSC')

varargout(1) = { [0 4 ; 4 0] };

varargout(2) = { [0 2 ; 2 0] };

varargout(3) = { [0 1 ; 5 0] };

elseif strcmpi(code, 'BBH')

error ('GENERATORS not available for BBH codes.');
```

```
elseif strcmpi(code, 'new')

varargout(1) = { [5 0 ; 3 5] };

varargout(2) = { [1 4 ; 4 1] };

varargout(3) = { [0 2 ; 2 0] };

end

% Create the Space Time TRELLIS Structure from the GENERATOR MATRICES.

%% Extract the no. of STATES and no. of outputs (Tx Antennas).

Nobits = size(varargin,2); % No. of bits per symbol = No. of Generator Matrices.

Mpq = 2^Nobits; % No. of constellation points (M-PSK or M-QAM).

if Nobits <= 0

error ('Number of Generator Matrices must be >= 1.');

end

tempNt = 0;

v_stage = zeros(1, Nobits); % Memory order per stage. v1, v2 .... vm;

for i = 1 : Nobits

v_stage(i) = size(varargin{i},1) - 1;

if i == 1

tempNt = size(varargin{i},2);

elseif tempNt ~= size(varargin{i},2)

error ('Number of columns of each Generator Matrix must be same.');

end

if ndims(varargin{i}) ~= 2

error ('Each Generator Matrix must be a 2D matrix.');

end

end
```

```
vTotal = sum(v_stage); % Memory order, v = Total no. of registers used.

N_t = tempNt; % No. of Tx Antennas.

States = 2^vTotal; % No. of STATES.

%% Create single GENERATOR Matrix, G.

numBr = Mpq * States; % Total no. of transition branches in trellis.

G = zeros(log2(numBr), N_t);

i = log2(numBr);

j = 1;

while i > 0

for k = (Nobits : -1 : 1)

G(i, :) = varargin{k}(j , :);

i = i - 1;

if i <= 0

break;

end

end

j = j + 1;

end

%% Create Trellis.

outputM = zeros(numBr, N_t);

for i = 1 : numBr

b = dec2bin(i - 1, log2(numBr)) - 48;

outputM(i,:) = mod((b * G), Mpq);

end

Trellis = outputM';
```

```
for i = 1 : N_t
varargout(i) = {(reshape((outputM(:,i)), Mpq, States))'};
end
th(new)
Eb = 1; % Energy per bit.
Criterion = 'Tc';
Es = log2(M) * Eb; % Energy per symbol.
SNRdB = 0:2:30; % SNR in dB.
SNR = 10.^(SNRdB./10); % SNR in linear scale.
N0 = Es ./ SNR; % Noise.
N_var = N0; % Noise Variance, SIGMA² = N0.
N_sd = sqrt(N_var); % Noise Standard Deviation, SIGMA.
Fr = 1000; % Number of FRAMEs.
Sy = 130; % Number of symbols per FRAME.
disp('Started...');
%% Generate Trellis.
if M == 4
[g1 g2] = Generators(M, N_States, N_Transmit, Criterion, Code);
[Tr Tr1 Tr2] = Generator_Trellis(g1, g2);
elseif M == 8
[g1 g2 g3] = Generators(M, N_States, N_Transmit, Criterion, Code);
[Tr Tr1 Tr2] = Generator_Trellis(g1, g2, g3);
end
%% Generate random binary data BITs.
Data_In = round(rand(1, Sy*m));
```

```
%% Make appropriate number of BITs ZERO.
zerosNeeded = m * (N_States/M);
if zerosNeeded <= length(Data_In)
Data_In(1 : zerosNeeded) = 0;
Data_In(end-zerosNeeded + 1 : end) = 0;
else
Data_In = zeros(1, zerosNeeded);
end
%% Convert Data BITs into equivalent SYMBOLs.
Data_In_Sy = zeros(1, (size(Data_In, 1) * size(Data_In, 2))/m);
bit_weights = (2*ones(m, 1)).^((m-1 : -1 : 0)');
for k = 1 : length(Data_In_Sy)
Data_In_Sy(k) = sum(bit_weights .* (Data_In( m*(k-1) + 1 : m*(k-1) + m)'));
%% Encode
%% Set Parameters.
NumberofTx = size(Tr,1); % Number of Tx Antennas.
States = length(Tr) / M; % Number of STATES.
m = log2(M); % Number of BITs per SYMBOL.
NSymbols = length(Data_In) / m; % Number of symbols to encode.
%% Convert input data BITs into equivalent SYMBOLs.
Convert = zeros(1, (size(Data_In, 1) * size(Data_In, 2))/m);
bitweights = (2*ones(m, 1)).^((m-1 : -1 : 0)');
for k = 1 : length(Convert)
Convert(k) = sum(bitweights .* (Data_In( m*(k-1) + 1 : m*(k-1) + m)'));
end
```

```
X = zeros(NumberofTx, NSymbols);

Mapper = reshape(1 : States*M, M, States)';

CState = 0; % Initial STATE is always ZERO.

for k = 1 : NSymbols

X(:, k) = Tr(:, Mapper(CState+1, Convert(k) + 1));

CState = mod(M*CState, States) + Convert(k);

end

%% Modulate.

modulated_X = Modulator(X, M, Scheme);

VITERBI algorithm.

% Syntax:

% [es_Symbols es_Bits] = Viterbi_Decoder(r, Tr, M, BM)

% Description:

% Inputs:

% r -> Recieved signal ( Fading + Noise ).

% Tr -> Output Matrix of the ENCODER TRELLIS.

% M -> Number of CONSTELLATION POINTS (M-PSK or M-QAM).

% BM -> Branch metrics for whole trellis

%% Set Parameters.

mm = log2(M); % Number of bits per symbol.

nStates = size(Tr, 2)/M; % Number of STATES.

NSymbols = size(r, 2); % Number of received SYMBOLs.

ES = zeros(1, NSymbols);

ES_Bits = zeros(1, length(ES) * mm);

%% VITERBI ALGORITHM (MAXIMUM LIKELIHOOD DECODING).
```

```
P_M = zeros(nStates, 1);

S_H = zeros(nStates, NSymbols);

for k = 1 : NSymbols

B_M = reshape(BM(k, :), M, nStates);

B_M = B_M + repmat(P_M', M, 1);

[P_M S_H(:, k)] = min(reshape(reshape(B_M, 1, nStates * M), nStates, M), [], 2);

S_H(:, k) = (nStates /M) * (S_H(:, k) - 1) + floor((0: nStates -1)/M)';

end

% Traceback.

for k = NSymbols : -1 : 2

ES(k-1) = S_H(ES(k)+1 , k);

end

ES = mod(ES, M);

%% INITIAL & FINAL STATE is always assumed to be ZERO.

ES(1 : nStates/M) = 0;

ES(end-nStates/M + 1 : end) = 0;

%% Convert SYMBOLs to BITs.

for k = 1 : NSymbols

ES_Bits(1, mm*(k-1)+1 : mm*(k-1) + mm) = int8(dec2bin(ES(k), mm)) - 48;

end
```