

**Phased Linear Array of Circular Aperture Antennas****Ahmed Hashim Abood****College of Medicine – Misan University****Abstract**

**In this paper the phased array consist of a circular aperture antennas are arranged in linear array to study the effect of the number of elements on the radiation pattern parameters.**

**The effect of distribution of current on each elements also studied to reduced the side lobe level.**

## Introduction

Several antennas are arranged in space and interconnected to produce a directional radiation pattern. Such configuration of multiple radiating elements are referred to as an array antenna, or simply, an array<sup>[1]</sup>. For application where higher gain or more directive radiation patterns are required, it is necessary to increase the size of antenna. Using an array of radiating elements can increase the antenna size. The radiated field from an array of a given point in space is the vector sum of the radiated fields from the individual elements. In other words, the far field of the array is determined by superposition<sup>[1]</sup>.

Elements of an array are dipoles, monopoles, loops, slots, microstrip patches, and horns are the most common types of array elements. The array antennas are classified according to<sup>[2]</sup>:

**Radiation pattern:** broadside, endfire, intermediate, omnidirectional, scanned, shaped beam, multiple beam, adaptive or constrained.

**Geometry:** linear, planar, circular, flat, three dimensional, or conformal.

**Elements:** dipoles, monopoles, loops, slots, microstrip patches, horns, spiral, helices, long periodic, monolithic, active, electrically small.

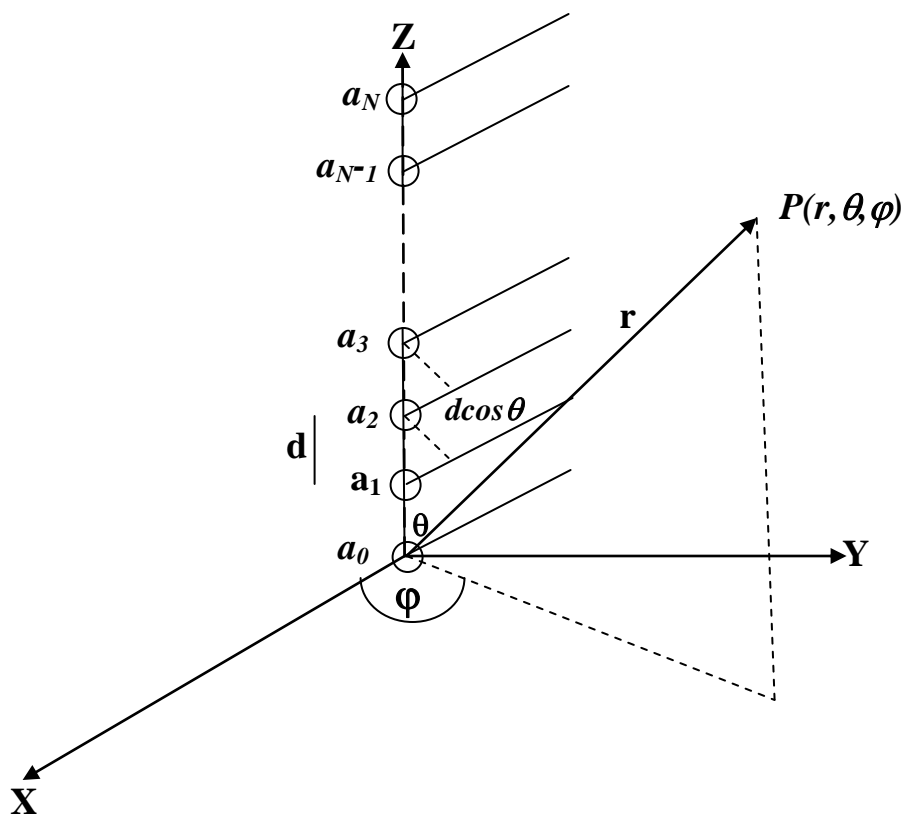
**Excitation:** uniform, binomial, Chebyshev, Fourier, ..... etc.

Phased array antennas consists of multiple stationary antenna elements, which are fed coherently by using variable phase or time delay control at each element to scan beam to given angles in space. Variable amplitude control is sometimes also provided for pattern shaping.

Beam steering is a concept achieved by controlling the phase or time delay between elements of an array using, as an example, microstrip patches antenna, because they are light and un expensive and easy to fabricate<sup>[3]</sup>, there are many of parametera affect on the radiation pattern of an array antenna such as the mutual coupling between the elements, the parameter is studied extensively by<sup>[4]</sup>. Beam steering achieved manually, electrically or electronically by fabricate the electrical or electronically circuit called beam forming circuit<sup>[5]</sup>.

A linear array is a set of radiating elements arranged on a line as illustrated in Fig

. (1) . It should be noted here that the formulation here is for equally spaced linear array .



**Fig . (1) : Linear array of equally spaced elements**

An antenna that has a physical aperture opening with a circular shape is said to be circular aperture, various forms of circular aperture antennas are encountered in practice. The radiation pattern of the circular aperture antenna is determined by using the sufficient value of aperture's diameter and current distribution function on the aperture, this antenna may be excited uniformly or non-uniformly distribution.

The combined electric field measured at a far field point P is computed as the product between the array factor and the element pattern,

$$E(P) = E(\text{one element})(\text{array factor}) \dots\dots\dots (1)$$

The array factor is general function of the number of elements, their spacing and their relative phases and magnitudes<sup>[6]</sup>.

In this paper the linear array consist of circular aperture as an elements is studied and applying the phased array.

**Theory:**

Consider liner array of circular aperture as depicted in Fig.(1). Let element  $a_0$  serve as a phase reference of the array. From the geometry. It is clear that an outgoing wave at the  $n$ <sup>th</sup> element lags the phase at the  $(n-1)$ <sup>th</sup> element by  $kdc\cos\theta$ , where  $k=2\pi/\lambda$ . The element factor is the circular aperture, the radiation field (electric field) is given by<sup>[6]</sup>,

$$E(\theta, \varphi) = \int_S E_a(\rho) e^{jk\vec{r}\cdot\vec{r}'} da \dots\dots\dots (2)$$

Where  $a$  denotes aperture, and

$$E_a(\rho') = \left[ 1 - \left( \frac{\rho'}{a} \right)^2 \right]^n \dots\dots\dots (3)$$

and

$$da = \rho d\rho d\varphi'$$

Where  $n$  is an integer represent the distribution type, as an example  $n=0$  refers to uniform distribution and  $n=1$  means parabolic distribution and so on.

To simplifying eq.(1), some mathematical manipulations are done as

$$\vec{r} = \sin\theta\cos\varphi\hat{x} + \sin\theta\sin\varphi\hat{y} + \cos\theta\hat{z}$$

$$\vec{r}' = x'\hat{x} + y'\hat{y}, \quad x' = \rho'\cos\varphi' \text{ and } y' = \rho'\sin\varphi'$$

So,

$$\vec{r}\cdot\vec{r}' = \rho'\sin\theta\cos\varphi\cos\varphi' + \rho'\sin\theta\sin\varphi\sin\varphi' = \rho'\sin\theta\cos(\varphi - \varphi')$$

Then eq.(1) become,

$$E(\theta, \varphi) = \int_0^a \left[ 1 - \left( \frac{\rho'}{a} \right)^2 \right]^n \rho' d\rho' \int_0^{2\pi} e^{jk\rho'\sin\theta\cos(\varphi - \varphi')} d\varphi' \dots\dots (4)$$

In case of uniform distribution (i.e.  $n=0$ ) the current distribution function is unity, one can get

$$E_a(\theta, \varphi) = \int_0^a \rho' d\rho' \int_0^{2\pi} e^{jk\rho'\sin\theta\cos(\varphi - \varphi')} d\varphi' \dots\dots\dots (5)$$

The second integral of  $\int_0^{2\pi}$  is solve by using identity of Bessel's function as

$$\int_0^{2\pi} e^{jkz\cos\xi} d\xi = 2\pi J_0(z) \dots\dots\dots (5-a)$$

Where  $J_0(z)$  is the Bessel function of the first kind of the order zero. Because of the circular symmetry over aperture, the electric field independent of  $\phi$ , hence,  $E_a(\theta, \phi) = E_a(\theta)$  and eq.(1), re-written as,

$$E_a(\theta) = \int_0^a \rho' J_0(k\rho' \sin\theta) d\rho'$$

$$E_a(\theta) = \pi a^2 \frac{2J_1(k\rho' \sin\theta)}{(k\rho' \sin\theta)} \dots\dots\dots (6)$$

In the general form (i.e.  $n > 0$ ) re-call that

$$\int_0^1 x [1 - x^2]^n J_0(bx) dx = \frac{2^n n!}{b^{n+1}} J_{n+1}(b) \dots\dots\dots(6-a)$$

Suppose,  $x = \rho'/a$  and  $b = k\rho' \sin\theta$ , eq.(5) is given by

$$E_a(\theta) = \frac{\pi a^2 2^{n+1} (n+1)! J_{n+1}(k a \sin\theta)}{n+1 (k a \sin\theta)^{n+2}} \dots\dots\dots$$

(7)

The array factor as shown in Fig.(1) is given by.

$$E(\theta) = \sum_{i=1}^N e^{j(i-1)kdsin\theta} \dots\dots\dots (8)$$

Expanding the summation in Eq.(8) yields,

$$E(\theta) = 1 + e^{jkdsin\theta} + \dots + e^{j(N-1)kdsin\theta} \dots\dots\dots (9)$$

The right side of Eq.(9) is a geometric series, which can be expressed in the form

$$1 + a + a^2 + a^3 + \dots + a^{N-1} = \frac{1-a^N}{1-a} \dots\dots\dots (10)$$

Replacing  $a$  by  $kdsin\theta$  yields,

$$E(\theta) = \frac{1 - e^{jNkdsin\theta}}{1 - e^{jkdsin\theta}} = \frac{1 - \cos Nkdsin\theta - jsin Nkdsin\theta}{1 - \cos kdsin\theta - jsin kdsin\theta} \dots\dots\dots(11)$$

$$|E(\theta)| = \left| \frac{\sin\left(\frac{Nkdsin\theta}{2}\right)}{\sin\left(\frac{kdsin\theta}{2}\right)} \right| \dots\dots\dots (12)$$

The maximum value of  $|E(\theta)|$  occurs at  $\theta = 0$  and it is equal to  $N$ . it follows that the normalized intensity pattern is equal to,

$$|E(\theta)| = \frac{1}{N} \left| \frac{\sin\left(\frac{Nkdsin\theta}{2}\right)}{\sin\left(\frac{kdsin\theta}{2}\right)} \right| \dots\dots\dots (13)$$

The total radiation at point P is

$$E(P) = (E_a(\theta) = \frac{\pi a^2 2^{n+1} (n+1)! J_{n+1}(ka \sin \theta)}{(ka \sin \theta)^{n+2}}) \left( \frac{1}{N} \left| \frac{\sin\left(\frac{Nkd \sin \theta}{2}\right)}{\sin\left(\frac{kd \sin \theta}{2}\right)} \right| \right) \dots \dots \dots (14)$$

Where N is the number of elements, d is the space between elements, and a is the radius of a circular aperture.

The main beam of an array can be steered electrically by varying the phase of current applied to each array element. Steering the main beam into the direction- $\sin \theta_o$  is accomplished by making the phase difference between the two adjacent elements equal to  $k d \sin \theta_o$ . In this case, the normalized radiation pattern can be written as,

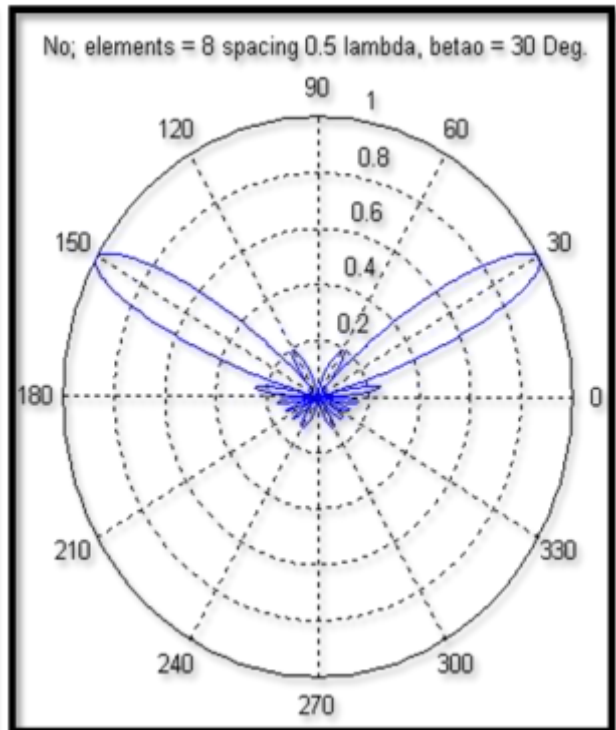
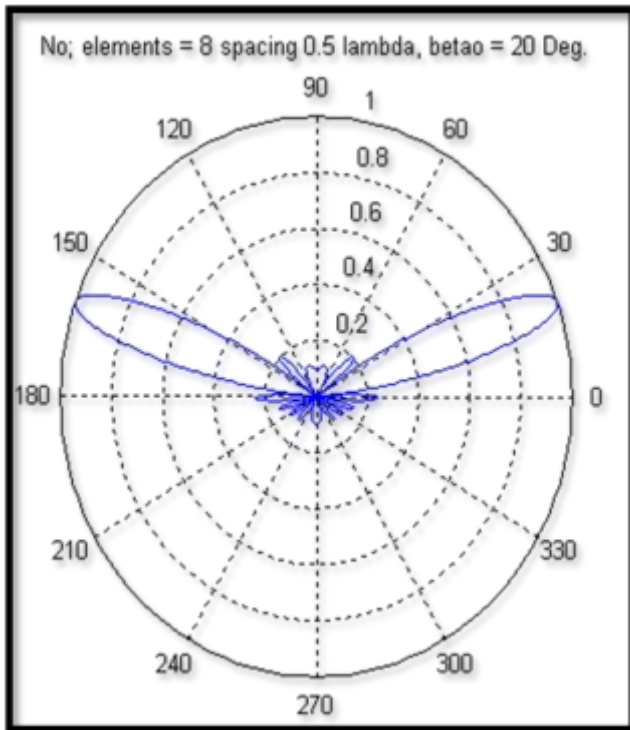
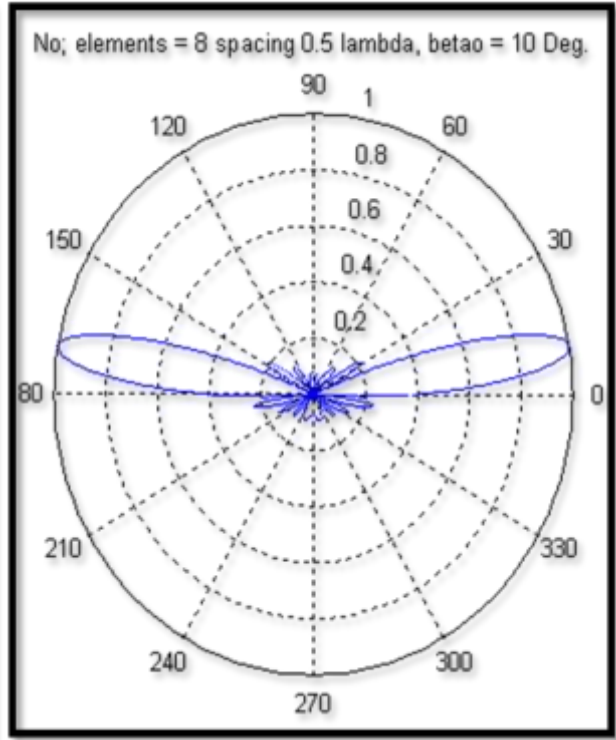
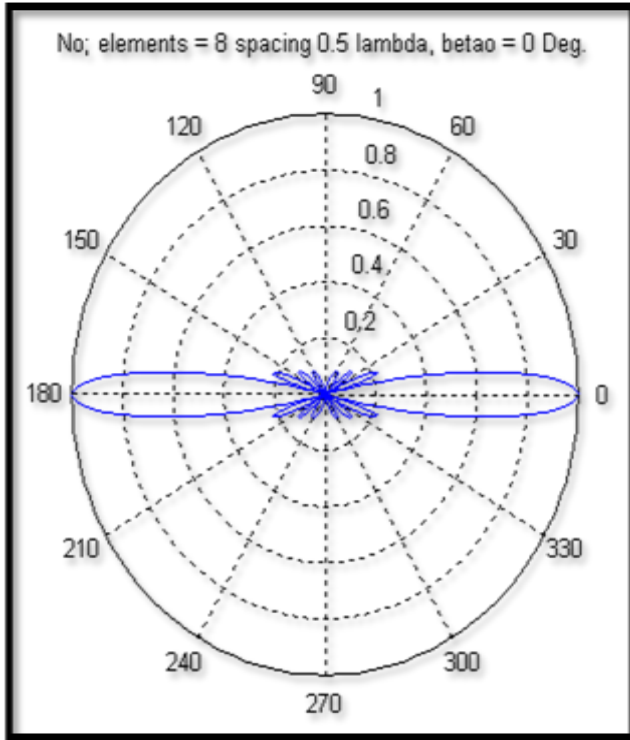
$$|E(\theta)| = \frac{1}{N} \left| \frac{\sin\left(\frac{Nkd}{2}(\sin \theta - \sin \theta_o)\right)}{\sin\left(\frac{kd}{2}(\sin \theta - \sin \theta_o)\right)} \right| \dots \dots \dots (15)$$

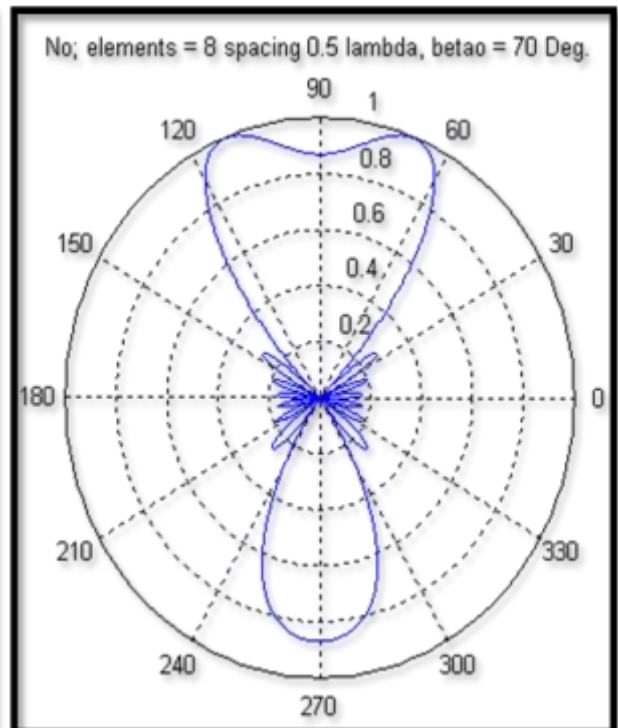
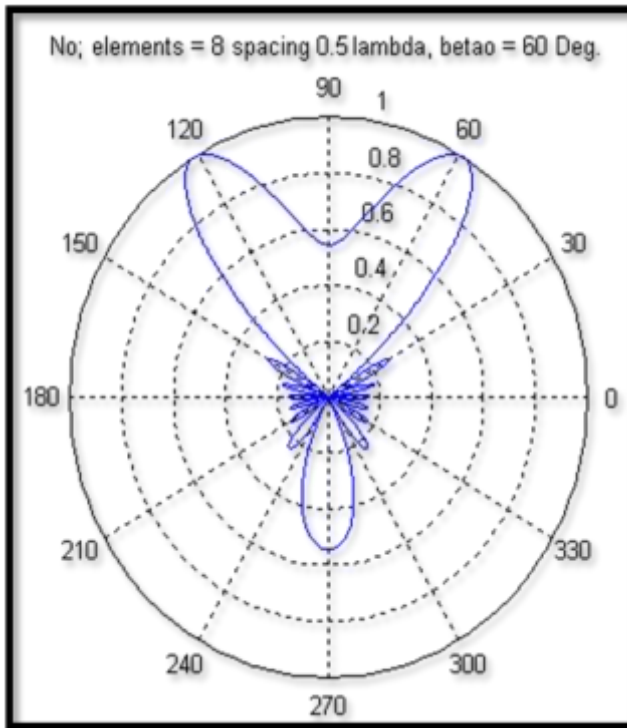
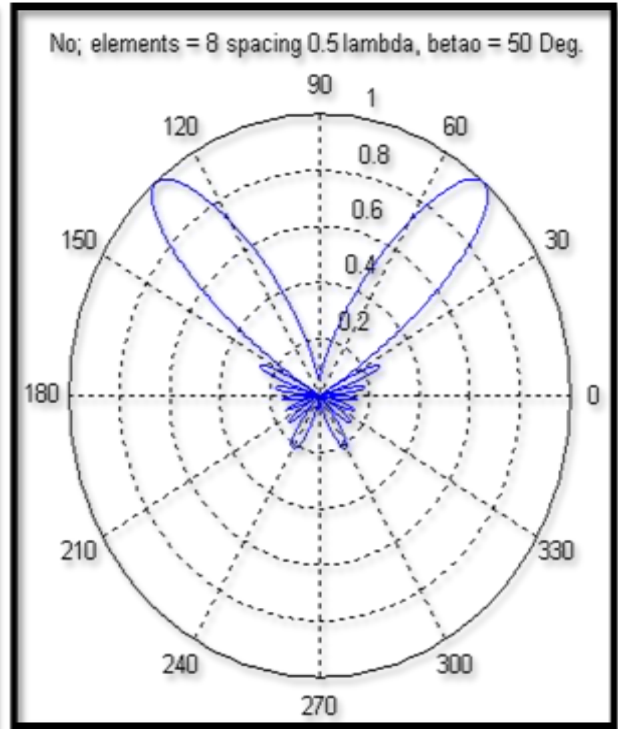
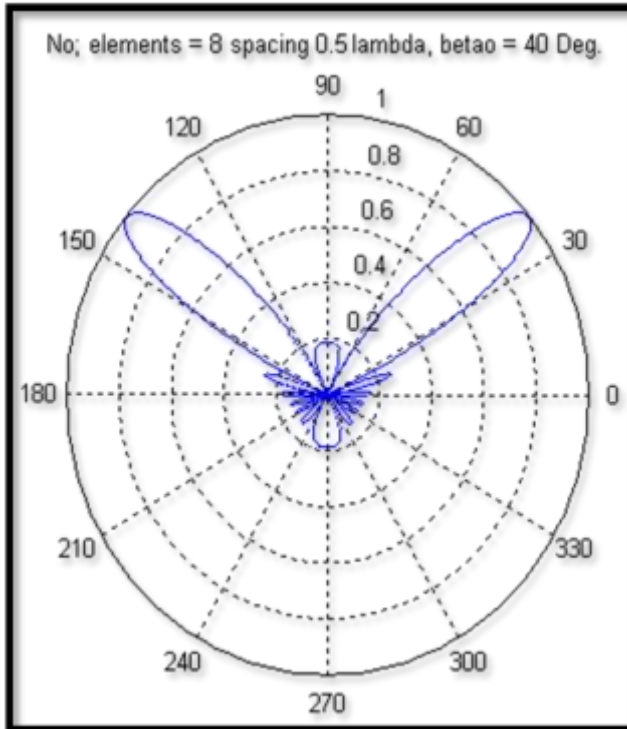
Where  $\theta_o$  is steering angle.

**Numerical Results**

After completing the mathematical treatment of the electromagnetic problem, the source code has been build with the help of MATLAB program the study the effect of the number of elements and steering angle.

A circular aperture of  $0.5\lambda$  diameter has been used with N=8 elements arranged along the z-axis with equally spaced of  $0.5\lambda$ . Fig.(2) shows the result of steered pattern from  $0^\circ$  to  $90^\circ$ .







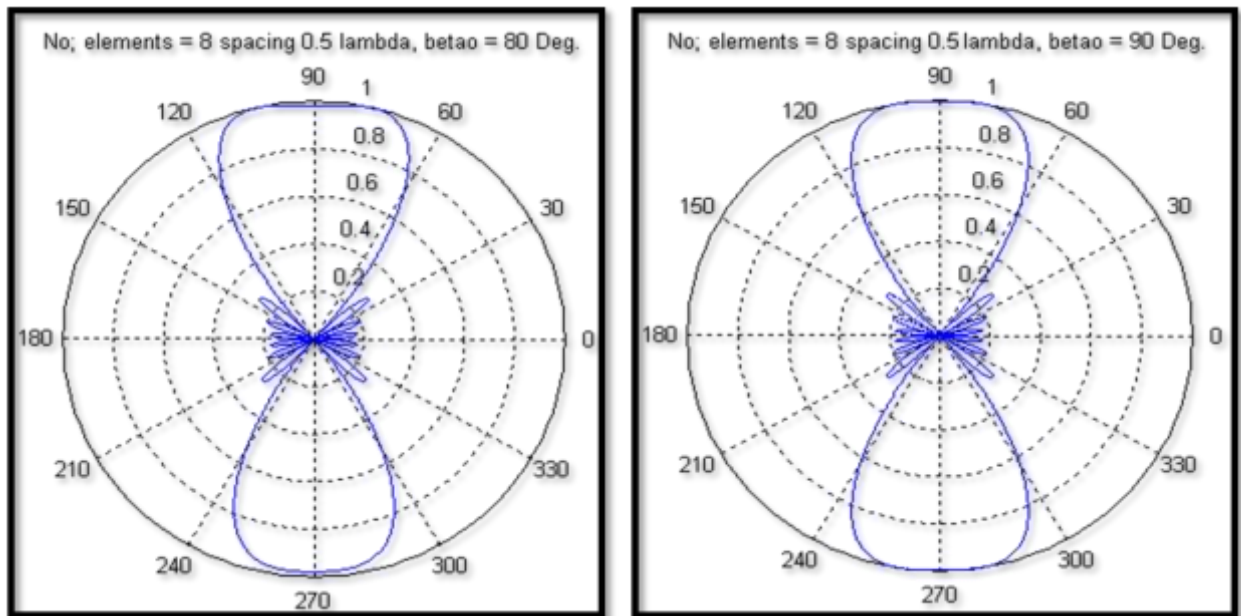
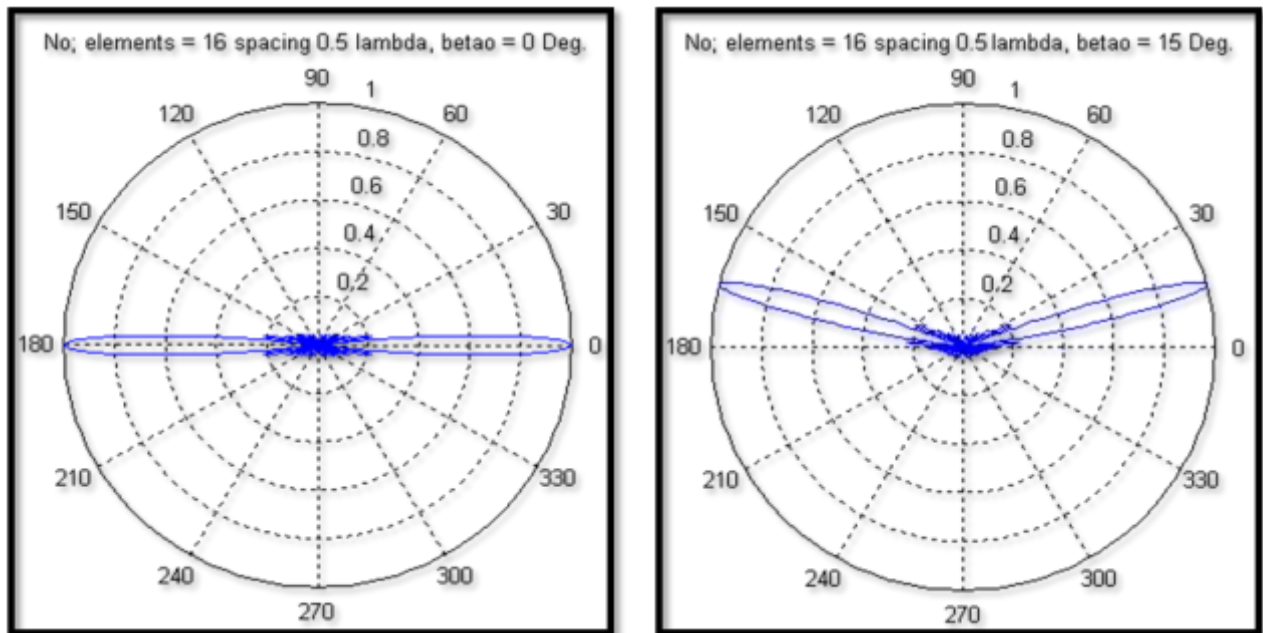
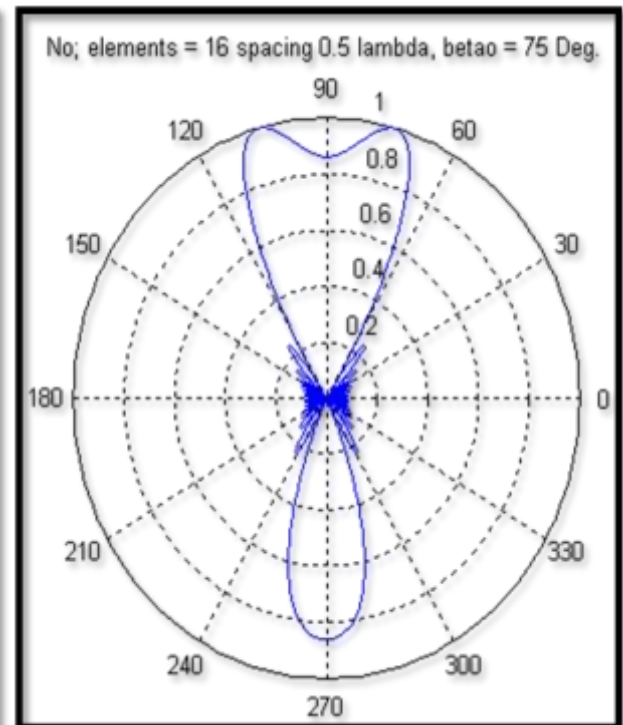
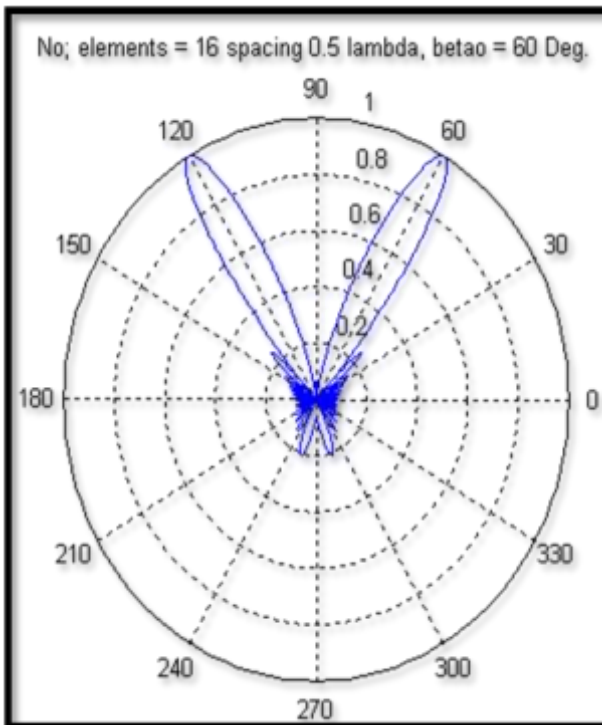
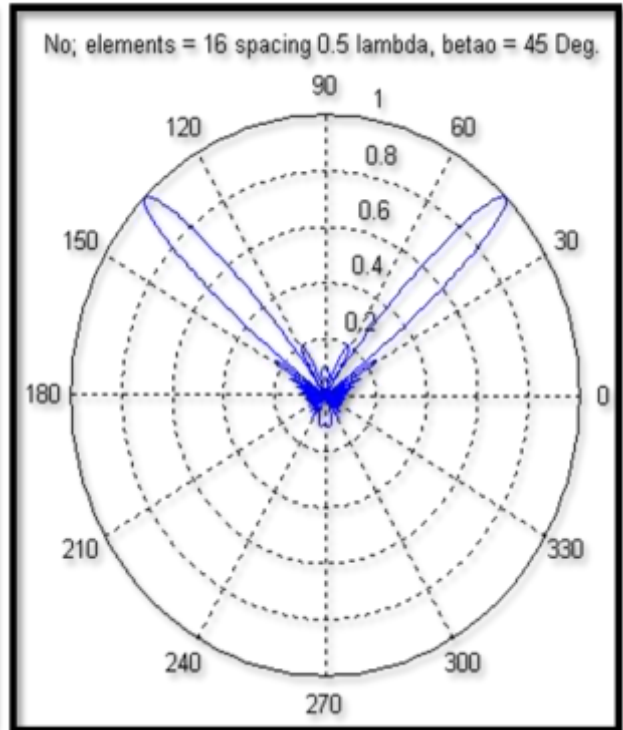
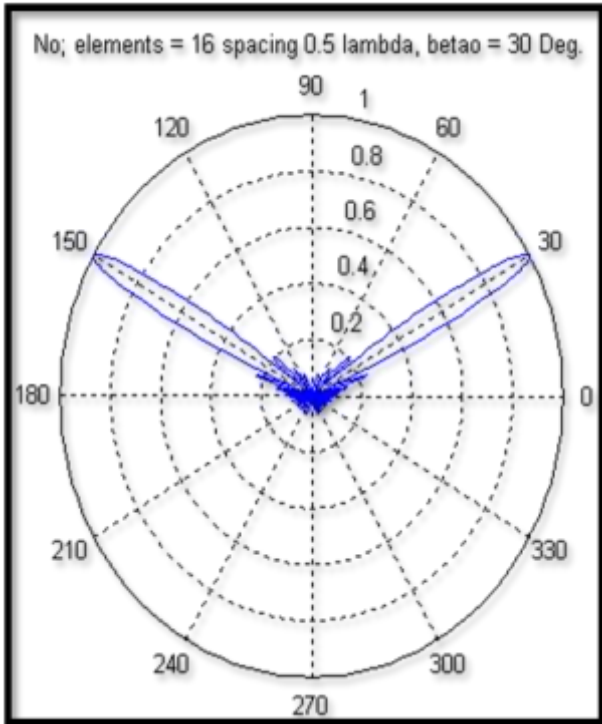


Fig.(2): Radiation pattern in polar coordinate of  $N = 8$ ,  $d = \lambda/2$ ,  $a = \lambda/2$ : (a)  $0^\circ$ , (b)  $10^\circ$ , (c)  $20^\circ$ , (d)  $30^\circ$ , (e)  $40^\circ$ , (f)  $50^\circ$ , (g)  $60^\circ$ , (h)  $70^\circ$ , (i)  $80^\circ$ , (j)  $90^\circ$

In case of increasing the number of elements, an array of 16 elements with of  $0.5\lambda$ . Fig.(3) shows the result of steered pattern from  $0^\circ$  to  $90^\circ$ .





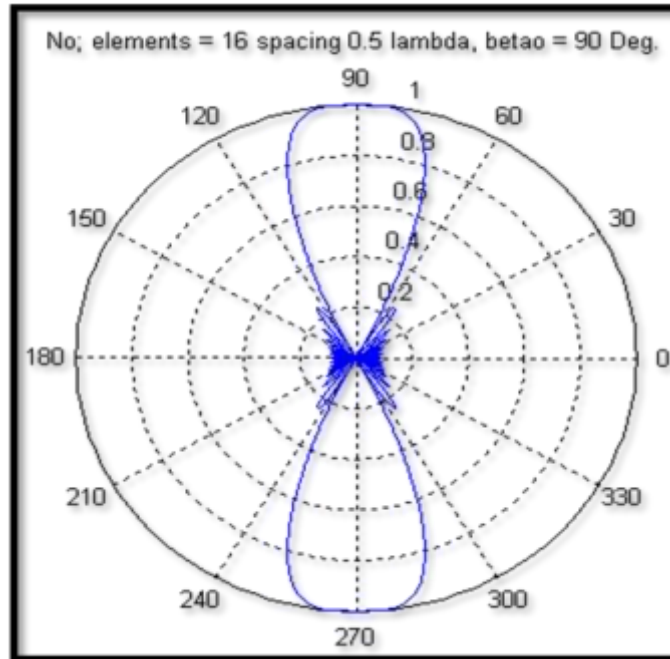


Fig.(3): Radiation pattern in polar coordinate of  $N = 16$ ,  $d = \lambda/2$ ,  $a = \lambda/2$ : (a)  $0^\circ$ , (b)  $15^\circ$ , (c)  $30^\circ$ , (d)  $45^\circ$ , (e)  $60^\circ$ , (f)  $75^\circ$ , (g)  $90^\circ$

## Conclusions

As shown from Fig.(2) and Fig.(3) the main lobe becomes narrow then the directivity is increased when the number of array element increased. Also, the array pattern is steered by controlling the phased of current distribution to each array element.

## References

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## مصفوفة هوائيات الفتحة الدائرية الخطية الطورية

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### الخلاصة

في هذا العمل المصفوفة الطورية المكونة من هوائيات الفتحة الدائرية المرتبة بشكل مصفوفة خطية دُرسَ تأثير عدد العناصر على معاملات هيكل الشعاع. تم دراسة تأثير دالة توزيع التيار على كل عنصر من عناصر المصفوفة وذلك لدراسة تقليل مستوى الفلقات الثانوية لهيكل الاشعاع.