University of Kent

Soliton Equations in Two Spatial Dimensions and Their Solutions

SOLITON EQUATIONS IN TWO SPATIAL DIMENSIONS AND THEIR SOLUTIONS

A THESIS SUBMITTED TO
THE UNIVERSITY OF KENT
IN THE SUBJECT OF
APPLIED MATHEMATICS
FOR THE DEGREE
OF DOCTOR OF PHILOSOPHY

BY
MOHAMMED JABBAR HAWAS ALLAMI

Abstract

This thesis concerns soliton equations in two spatial dimensions. Camassa and Holm derived a model of shallow water wave which continues to attract attention in the light of the remarkable wealth of mathematical and physical properties of its solutions. More recently, Kraenkel and Zenchuk introduced an extension of Camassa and Holm's equation to two spatial dimensions, with three fields, and derived it from a Lax pair. However, finding solutions of this model is more challenging.

In order to understand and find solutions of the $(2+1)$-dimensional Camassa-Holm (CH) equation, it turns out that we have to study and analyse other soliton equations in two spatial dimensions, together with their hierarchies. To begin with, we consider a squared eigenfunction symmetry of the Kadomtsev-Petviashvili (KP) hierarchy, this results in a system of equations in $(2+1)$-dimensions.

One of the main tools employed is Hirota's bilinear method. Using Hirota's perturbation technique, one-, two- and three-soliton solutions of the KP squared eigenfunction flow directly constructed, and numerical plots are presented. An ansatz for multisoliton solutions is proposed and proved by induction that it is correct. The soliton formula is shown to be equivalent to the formulae found by Freeman, Gilson and Nimmo, who derived Wronskian formulae for multisoliton solutions. The travelling wave solution is calculated for the three fields and Painlevé analysis is also

applied to the KP squared eigenfunction flow. Rational solutions for the KP squared eigenfunction flow are also generated.

Next we move on to deal with the modified Kadomtsev-Petviashvili (mKP) squared eigenfunction flow. Using the link, via a Bäcklund transformation, with the KP squared eigenfunction flow, we acquire one-, two- and three-soliton solutions directly. We also present an ansatz for the multisoliton solution of the mKP squared eigenfunction flow. The link, via reciprocal and Bäcklund transformations, between the Lax Pair of the mKP squared eigenfunction flow and $(2+1)$-dimensional CH system is crucial for securing the solutions for the latter system.

Finally, we consider solutions of the $(2+1)$-dimensional CH system. We obtain one-, two-, three-soliton solutions for the system. The solutions are implicit, and the derivation is much simpler than results of Zenchuk and collaborators. Additionally, the travelling wave solution for the $(2+1)$-dimensional CH system is computed.