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Al-Mustansiriyyah University

College of Science

Department of Mathematics

# ON FULLY STABLE ACTS

A Thesis

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# ABSTRACT

Let  $S$  be a monoid and  $M_S$  is a right  $S$ -act. We say that a subact  $N_S$  of  $M_S$  is stable if  $\theta(N) \subseteq N$  for each  $\theta \in \text{Hom}_S(N, M)$ .  $M_S$  is called fully stable if every subact of  $M_S$  is stable. A large part of this work is devoted to the study of the properties of the class of fully stable acts, and the relations between this class and other well studied classes of acts, like quasi-injective acts, acts satisfying Baer's criterion and multiplication acts. It is shown, for example, that every fully stable act is quasi-injective. but we know in module theory the last statement need not be true in general; also it is shown that when an  $S$ -act  $M_S$  is fully stable, we have that, every finitely generated subact of its will be satisfied the double annihilator condition.

It is shown that the endomorphisms monoid of fully stable acts is commutative. This suggests the following question. How can one characterize  $S$ -acts in which the monoid of endomorphisms of every subact is commutative? This question was asked in the special case on rings by W. Vasconcelos, and was given an answer by S. Alamelu and S.H. Cox, Jr. independently. Also M.S. Abbas was asked this question in module theory and he was given an answer. In this thesis we give a partial answer to the general question. In fact, we show that if  $S$  is a right inverse monoid,  $M_S$  is a strongly faithful act. Then  $\text{End}_S(N) \cong S/\mathcal{R}_S(N)$  for each subact  $N$  of  $M$  if and only if Baer criterion holds for  $M$ .



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بإشراف

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