Z^{*}-SMALL SUBMODULES AND Z^{*}-HOLLOW MODULES

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ABSTRACT. This work introduces a new class of submodules which we call Z^{*}-small, Z^{*}-maximal and Z^{*}-radical submodules. We show that there is no general relation between the maximal and Z^{*}-maximal submodules. In contrast, Z^{*}-small and Z^{*}-radical submodules are generalizations of the small and radical submodules. Also, this work introduces a new class of modules called Z^{*}-hollow and Z^{*}-semihollow as a generalization of the hollow and semihollow modules and shows no general relation between e^{*}-essential small and Z^{*}-small. Further, this work studies some properties of those concepts, such as the finite sums, direct sums and images of those concepts. Finally, this work investigates submodules which are not Z^{*}-small, Z^{*}-maximal, Z^{*}-radical, Z^{*}-hollow, or Z^{*}-semihollow.

1. Introduction

Throughout this paper all rings are commutative with identity and all modules are unitary right R-modules. A module M is called cosingular when $Z^{*}(M) = \{x \in M | xR \text{ is a small in } E(M)\} = M$, where $E(M)$ is the injective envelope of M. For a submodule A of module M, $Z^{*}(A) = A \cap Z^{*}(M)$. Cosingular submodules are closed under submodules, direct sums, homomorphic image, and every $\mathbb{Z}$-module is cosingular. For more details about cosingular submodules, see [1]. If $C + C' = M$, then $C' = M$, a submodule C of module M is said to be small, see [2] and [3]. Jacobson radical submodule of M is the total of all small submodules of M [3]. If each proper submodule is small, a nonzero module M is said to be hollow, see [4] and [5]. A nonzero module M is considered semihollow if each proper finite generated submodule is small, see[5] and [6]. Many authors generalized small submodule, see [7, 8, 9].

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