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Department
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EFFECT OF COMPRESSION, IMPACT AND SLIPPING ON ROLLING CONTACT FATIGUE AND SUBSURFACE MICROSTRUCTURAL DAMAGE

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ABSTRACT

Rolling Contact Fatigue (RCF) manifests itself in different engineering applications such as bearings, gears, railway tracks, and cams. In Wind Turbine Gearboxes (WTGs), which are designed to be in service for 20 to 25 years, the service life of their bearings is often below their design life despite depending on advanced technologies and standards in the gearbox design. This premature bearing failure occurs by flaking, mainly in the bearing inner races. Furthermore, bearing service life cannot be precisely predicted despite many life prediction models and using advanced analyses for the gearbox design. This premature bearing failure increases the wind energy cost due to unplanned maintenance and early replacement. In addition to that the main causes and mechanisms of this premature failure have not been completely understood. This has motivated an increasing investigation in this field due to the scientific and economic impacts.

There are a considerable number of factors affecting premature bearing failure. Three parameters which are widely reported as the main causes were investigated in this study which are contact pressure, slipping and impact loading. Two failed planetary bearings from a multi-megawatt wind turbine gearbox were investigated first to evaluate their surface and subsurface damage features and to estimate the parameter testing levels. A new test rig that can apply impact loading in combination with compression and slipping was designed. Two contact discs made from bearing material (AISI 52100 steel) were used in this test rig. Sixteen tests are designed using Minitab software for design of experiments using a line contact with flat test discs with stress concentration at the contact edges because stress concentration cannot be avoided in roller bearings, which is the type usually used in the turbine gearboxes. The effect of the key parameters on the test disc life in terms of the number of cycles to failure and subsurface damage features distribution at the microscale level were analyzed. These distributions were correlated with the subsurface stress distribution within the contact region of the bearings and the test discs to predict which stress type was responsible for each damage type. Further tests were also designed and conducted to avoid the stress concentration using fully crowned disc profiles to investigate the effect of the key parameters on disc life. The investigation led to the suggestion of a new simple damage estimation model depending on the contact stress levels and the number of cycles under each level.

The results of the qualitative and quantitative investigation confirm the considerable role of contact stress followed by slipping and then impact loading. Despite impact-loading affecting damage as contact pressure, it reveals two different effects on the subsurface damage features. The first was observed in shallow regions as internally cracked inclusions within the disc material while, the second by introducing damaged inclusions in deeper regions. The individual and interactive effects of impact loading with the other two parameters were found to be different from the compression loading effects. The correlation of damage distribution with the subsurface stress distributions confirms the postulation of Von-Mises and maximum shear stresses as being the main contributors to damage initiation and propagation. The suggested life prediction model was tested by using the test results and real wind turbine operating data of SCADA for two years in addition to using the average annual wind speed distribution. The results of this model were very close to the reported wind turbine gearbox bearing life. More testing is still required to confirm the reliability of this simple and applicable model. The metallographic investigations confirmed that the subsurface microcracks were often not associated with non-metallic inclusions, but may be started from the voids associated with the material carbides is a significant damage initiation source in addition to that of non-metallic inclusions which have been widely reported in the previous studies. Furthermore, Slipping Ratio (SR) is found to have more effect under low contact loading levels. A considerable role of impact loading in the butterfly wing damage feature is postulated. Moreover, the Intrusion/Extrusion and dislocation damage mechanism theories may be more suitable for describing overloading fatigue damage.

The novelty of this study is that it is the first study of the rolling contact fatigue life and the behaviour of the bearing steel under impact-loading in combination with compression and sliding to investigate the individual and interactive effects of these parameters on the trend of fatigue life variation by developing a new test rig. A new bearing life prediction methodology is proposed, and the results showed a lower percentage of errors in the turbine bearing predicted life compared with the standards currently use. More tests and metallographic investigations are recommended to investigate the effect of lower and higher contact stress levels and impact loading to understand the effect of these parameters on the initiation and propagation of the subsurface damage features.

PUBLICATIONS AND PRESENTATIONS

Publications

- ❖ Jasim H Al-Bedhany and Hui Long, “Microscopic investigation of subsurface initiated damage of wind turbine gearbox bearings,” J. Phys.: Conf. Ser. 1106 012029, 2018.
- ❖ Tahseen Ali Mankhi, Stanisław Legutko, Jasim H AL-Bedhany and Abdulmuttalib A Muhsen, “Selecting the Most Efficient Bearing of Wind Turbine Gearbox Using (Analytical Hierarchy Process) Method “AHP”, International Conference on Sustainable Engineering Techniques, Iraq, March 2019.

Presentations

- ❖ “Investigation of premature failure of wind turbine gearbox bearings,” Energy 2050, Energy Research Symposium (ERS), Grantham Centre, the University of Sheffield, UK, 2017.
- ❖ “Microscopic investigation of subsurface initiated damage of wind turbine gearbox bearings”, MPSVA conference, Clare College, University of Cambridge, Cambridge, UK, 2018.

Poster presentations

- ❖ “BEARING WHITE ETCHING CRACK DAMAGE AND LIFE PREDICTION METHOD,” a poster presented in the Ph.D. poster Event at the University of Sheffield 2017.

DEDICATION

To my homeland (IRAQ) the dearest than all. To every drop of blood spilled by the Iraqi martyrs and any others injured during the battle against ISIS (Terrorism State). To my university (Misan), its Engineering college and everybody who supported me during the study.

To the spirit of my father, to my mother and my brother Ahmed.

To my wife and my children Hasan, Aya, Retaj, Zahraa, and Ahmed.

To my friends who care about me and hope me to succeed.

I hope I will be as you expect, and I promise you that, I shall continue doing my best to serve humanity.

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NOMENCLATURES

Unless otherwise stated, all units used through this thesis are SI.

| | |
|--|---|
| σ | Stress (Pa) |
| P_o | Maximum contact pressure (Pa) |
| a and b | Half of the contact width (m) |
| τ , τ_{xy} , τ_{max} | Shear, orthogonal and unidirectional shear stresses (Pa) |
| $\sigma_1, \sigma_2, \sigma_3$ | Principal stresses (Pa) |
| σ_y | Yield stress (Pa) |
| z | Depth beneath the contact surface (m) |
| Z_{rol} | Number of bearing roller |
| L_{10} | Rated bearing life with 90% reliability in million cycles (Cycle) |
| c | Dynamic load capacity (N) |
| p_{eq} | Equivalent radial load (Pa) |
| n | Number of cycles |
| N, N_f | Number of cycles to failure |
| e | Life estimation empirical exponent |
| D | Damage |
| x_k | Variable depends on loading level |
| r | Cyclic ratio (number of rotating cycles to number of cycles to failure) |
| A and B | Constants |
| w | Exponent depend on fatigue stress amplitude |
| α | The fraction for crack initiation |
| β | The fraction of crack propagation |
| N_i, N_{II} | Number of cycles for crack initiation and propagation respectively |
| m and n | Contact parameters |

| | |
|------------------|---|
| s | Bearing raceway survival probability (%) |
| L_f | Fatigue life (Cycle) |
| a_o and a_f | Initial and final crack lengths (m). |
| E | Modulus of elasticity (GPa) |
| ν | Poisson's ratio |
| G | The gear ratio of the turbine gearbox |
| P_{gen} | Power of turbine generator (W) |
| H | Number of rows in planetary bearings |
| ω_{gen} | Rotating speed of the turbine generator (rad/s) |
| η | Overall drivetrain efficiency |
| R_D | Ring gear diameter in planetary stage (m) |
| S_D | Sun gear diameter in planetary stage (m) |
| T | Torque (N.m) |
| R_a | Arithmetic surface roughness |
| HV | Vickers hardness |
| HRC | Rockwell Hardness |
| U_{ud}, U_{ld} | The linear velocities of the upper and lower test disc respectively (m/s) |
| SR | Slipping ratio |
| AR | Aspect ratio |
| T_1, T_2 | Upper and lower test disc thickness respectively (m) |
| R_1, R_2 | Upper and lower test disc radii respectively |
| ρ | Density (kg/m ³) |
| φ | Dynamic viscosity (cSt) |
| h_f | Lubricant film thickness (μm) |
| \mathcal{K} | kinematic viscosity (cP) |
| ϑ | Pressure-viscosity coefficient |
| μ | Friction coefficient |

| | |
|-----------------------|---|
| V_{cam} | Tip cam velocity (m/s) |
| N_{cam} | Cam rotational speed (rpm) |
| R_c | Cam tip radius (m) |
| ω_{il} | Impact lever rotational speed (rad/s) |
| V_{imp} | Impact velocity (m/s) |
| M_{eff} | Effective impact mass (kg) |
| d_g | The distance between the radius of gyration and impact lever center (m) |
| I | Angular mass moment of inertia (kg.m ²) |
| L | Length (m) |
| k_s | Stiffness parameter (N/m) |
| E_{imp} | Impact energy (J) |
| F_i | Impact force (N) |
| p_{oi} | Maximum contact pressure due to impact (Pa) |
| μ_{mean} | Mean value of the output |
| β_{input} | Effect of the input factor |
| ε_{noise} | Error due to noise |
| SD | Standard deviation |
| β | Scale Parameter |
| \aleph | Shape Parameter |
| $F_{(pr)}$ | probability fraction |
| t_n | Test number |
| N_T | Total number of tests |
| t | Time (s) |

ABBREVIATIONS

| | |
|-------|--|
| RCF | Rolling Contact Fatigue |
| WTGs | Wind Turbine Gearboxes |
| O&M | Operating and Maintenance |
| WT | Wind Turbine |
| WE | Wind Energy |
| RE | Renewable Energy |
| LSS | Low-Speed Shaft |
| PP | Planetary Pin |
| HSS | High-Speed Shaft |
| MSS | Medium Speed Shaft |
| UW | UpWind |
| DW | Down Wind |
| WSF | White Structure Flaking |
| DEAs | Dark Etching Areas |
| WTGBs | Wind Turbine Gearbox Bearings |
| WEAs | White Etching Areas |
| WEBs | White Etching Bands |
| WECs | White Etching Cracks |
| SR | Slipping Ratio |
| R/S | Rolling/Sliding |
| SCADA | Supervisory Control And Data Acquisition |
| WTs | Wind Turbines |
| LRM | Light Reflection Microscope |
| OD | Over-rolling Direction |
| WL | White Layer |
| DZ | Deformed Zone |

| | |
|--------|--|
| LEDS | Low Energy Dislocation Structure |
| FHA | Frictional Heat Accumulation |
| LEAs | Light Etching Areas |
| LAB | Low Angle Band |
| HAB | High Angle Band |
| FE | Finite Element |
| EHL | Elastohydrodynamic Lubrication |
| NREL | National Renewable Energy Laboratory |
| SRCR | Single-Row Cylindrical Roller bearing |
| DRCR | Double-Row Cylindrical Roller bearing |
| SEM | Scanning Electron Microscope |
| EDX | Energy Dispersive X-ray analysis |
| WD | Working Distance |
| HV | Vickers Hardness |
| AISI | American Iron and Steel Institute |
| HRC | Rockwell Hardness |
| SRACBs | Single Row Angular Contact Ball Bearings |
| DOE | Design Of Experiment |
| SD | Standard Deviation |
| PDF | Probability Density Function |
| EFDA | Energy Fraction of Damage Accumulation |
| P-M | Palmgren-Miner theory |
| CTE | Coefficient of Thermal Expansion |
| DLDR | Double Linear Damage Rule |
| TF | Transfer Function |
| LEFM | Linear Elastic Fracture Mechanics |
| AR | Aspect Ratio |

| | |
|-----|-------------------------|
| PS | Probability of Survival |
| COD | Crack Opening Distance |

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