

## Abstract

In this thesis, we study general cocycles of dynamical systems in topological, measurable and smooth (differentiable) settings. Dynamical systems are viewed here as given by actions of a discrete, topological, measurable or Lie group on a set, topological space, measurable space or smooth manifold respectively depending on the given geometrical setting to be considered. We will mostly concentrate on the topological and smooth settings in this thesis, but will comment about the necessary alterations in the discrete and measurable setting. Cocycles are functions on the Cartesian product of the spaces and groups involved with values in an abelian group depending again on the given geometrical settings. A main task of this thesis is to interpret these cocycles as general cohomology classes of certain action groupoids, which decode the dynamical system. Similarly, we show that cohomology classes of action groupoids associated to dynamical systems can be viewed as cocycles. The action groupoids which arise out of the dynamical systems and the given geometrical setting are discrete groupoids, topological groupoids, measurable groupoids or Lie groupoids. We will introduce a very general groupoid cohomology and homology theory with values in vector bundles and discuss its basic properties generalising group cohomology and singular cohomology. Furthermore, we will study extensions of dynamical systems via cocycles and interpret these as low-dimensional cohomology classes. Some low-dimensional homology and cohomology groups are calculated. Finally, we interpret cocycle cohomology classes as cohomological obstructions for extending dynamical systems following a suggestion by Tao.